

1. (i) ~~Flow in a pipe~~ Pressure gradient is constant
 flow is steady
 flow is uniform

(ii) ~~Flow in a pipe~~ Diameter of the pipe
~~Flow in a pipe~~ Velocity of the flow
~~Flow in a pipe~~ Viscosity of the fluid
 density of the fluid passing through the pipe

| (iii) Hydrofoil | Aerofoil |
|---|---|
| <ul style="list-style-type: none"> A wing attached to the hull of a ship that raises it out of the water travelling at speed and thus reduces drag | <ul style="list-style-type: none"> Structure with curved surfaces designed to give the most favourable ratio of lift to drag in flight |
| <ul style="list-style-type: none"> A lifting surface that operates in water | <ul style="list-style-type: none"> A wing of an aircraft |

(b) $\mu = 0.9 \text{ centipoise}$ $b = 10 \text{ mm} = 0.01 \text{ m}$ $y = ?$ $u = 1 \text{ ms}^{-1}$ $du = 6 \text{ mm}$ $dp = 60 \text{ N/m}^2$

(i) Velocity distribution

$$u = \frac{\mu y}{b} \left[\frac{1}{2\mu} \left(\frac{dp}{dx} \right) (by - y^2) \right]$$

$$= \frac{1 \times y}{0.01} \left[\frac{1}{2 \times (0.9 \times 10^{-3})} \left(\frac{-60 \times 10^3}{60} \right) \cdot (0.01y - y^2) \right]$$

$$= 100y \cdot [-5560y + (5.56 \times 10^5 y^2)]$$

$$= -5560y + (5.56y^2 \times 10^5)$$

(ii) Flow rate, $q = \frac{ub}{2} - \frac{b^3}{12\mu} \left(\frac{dp}{dx} \right)$

$$= \frac{1 \times 0.01}{2} - \left[\frac{(0.01)^2}{12(0.9 \times 10^{-3})} \cdot \left(\frac{60 \times 10^3}{60} \right) \right]$$

$$= 0.005 - [9.259 \times 10^{-5} \times 10^3] = 0.005 + 9.259 \times 10^{-2}$$

$$= 0.005 + 0.0926 = 0.0976 \text{ m}^3/\text{s}$$

(iii) Shear stress distribution, $\tau = \frac{\mu u}{b} - \left[\frac{1}{2} \left(\frac{dp}{dx} \right) \cdot (b - 2y) \right]$

$$= \frac{0.9(10^{-3})}{0.01} - \left[\frac{1}{2} \left[-10^3 \right] \cdot (0.01 - 2y) \right]$$

$$= 0.09 - [0.5 \times -10^3 \times (0.01 - 2y)]$$

$$= 0.09 - [-5 + 10^3 y]$$

$$= 5.09 - 1000y, \quad y = 0.01$$

$$= -4.91$$

(2) $\mu = 0.9 \text{ Ns/m}^2$, $\theta = 45^\circ$, $\rho = 1260 \text{ kg/m}^3$, $u = -1.5 \text{ m/s}$, $b = 0.01 \text{ m}$, $P_1 = 250 \text{ kNm}^{-2}$, $P_2 = 80 \text{ kNm}^{-2}$

Solution

$$\frac{dp}{dx} = -128.95 \times 10^3 \text{ Nm}^{-2}$$

$$P_1' = P_1 + \rho gh = 250 \times 10^3 + (1260 \times 9.81 \times 1) = 2626 \times 10^3 \text{ Nm}^{-2}$$

$$P_2' = P_2 + \rho gh = 80 \times 10^3 + (1260 \times 9.81 \times 0) = 80 \times 10^3 \text{ Nm}^{-2}$$

$$\Delta P = 262.6 \times 10^3 - 80 \times 10^3 = 182.6 \times 10^3 \text{ Nm}^{-2}$$

(i) Velocity distribution

$$u = \frac{U_y}{b} \left[\frac{1}{2u} - \left(\frac{dp}{dx} \right) \cdot (by - y^2) \right]$$

$$= \frac{-1.5}{0.01} y - \left[\frac{1}{2(0.9)} \cdot (-128.95 \times 10^3) (0.01y - y^2) \right]$$

$$= -150y - [0.556 \cdot (-128.95 \times 10^3) (0.01y - y^2)]$$

$$u = 566.96y + (7.17 \times 10^4)y^2$$

(ii) Shear stress distribution, $u = 566y + (7.17 \times 10^4)y^2$

$$\frac{du}{dy} = 566 + 1.43 \times 10^5 y$$

$$\tau = \mu \left(\frac{du}{dy} \right) = 0.9 (566 + 1.43 \times 10^5 y)$$

$$\tau = 509.4 + 1.287 \times 10^5 y$$

(iii) Max velocity

U_{max} occurs at $\frac{du}{dy} = 0$

$$0 = 566.4 + 1.43 \times 10^5 y$$

$$y = \frac{-566.4}{1.43 \times 10^5} = -3.958 \times 10^{-3} \text{ m}$$

therefore

$$U_{\text{max}} = 566.96(y) + (7.17 \times 10^4)(y^2)$$

$$= -2.24 + 1.12$$

$$= -1.12 \text{ (in the opposite direction of the flow)}$$

(iv) $\tau = 509.4 + 1.287 \times 10^5 (-3.958 \times 10^{-3})$

$$= 509.4 + (-509.39)$$

$$= 0.01 \text{ Nm}^{-2}$$