

1a) Three Conditions of Cattle flow

- ① The flow rate
- ② The velocity
- ③ Shear stress

(ii) Conditions used to determine the nature of flow

- ① The Density (ρ)
- ② The flow velocity (V)
- ③ Fluid viscosity (μ)
- ④ The pipe diameter (D)

(iii) Aerofoil

→ is the cross sectional shape of a wing, blade or sail

→ It has a shape of the wing of an air craft

→ This is an air structure shaped to produce lift when moving in the air

Hydrofoil

is an attachment to the hull of a ship that raises the ship out of water traveling at speed

It can be artificial such as a rudder in a boat, submarine and surfboard fin

used in lifting element, the hydrofoil best the upward force.

1b) Viscosity of the liquid $\mu = 0.9 \text{ Centi poise} = 0.009 \text{ poise}$

Convert poise to SI unit.

$$10 \text{ poise} = 1.0 \text{ Ns/m}^2$$

$$0.009 \text{ poise} = \frac{0.009 \times 1.0}{10} \text{ Ns/m}^2$$

$$\text{Viscosity } \mu = 0.0009 \text{ Ns/m}^2$$

Distance b/w the plates (b) = 10 mm = 0.01 m

velocity of the upper plate (u) = 1 m/s

Pressure difference b/w the section 60m apart = 60 kPa/m²

$$\therefore \left(\frac{-\partial p}{\partial x} \right) = \frac{60 \times 10^3}{60} = 10^5 \text{ N/m}^2/\text{m}$$

① Velocity distribution

$$u = \frac{u}{b} y + \frac{1}{2\mu} \left(\frac{-\partial p}{\partial x} \right) (by - y^2)$$

$$= \frac{1}{0.01} y + \frac{1}{2 \times 0.0009} (10^5) (0.01) y - y^2$$

$$= \frac{1}{0.01} y + 555555.5556 (0.01y - y^2)$$

$$= \frac{1}{0.01} y + 5555.555556y - 555555.5556y^2$$

$$= \frac{y}{0.01} + 5555.555556y - 555555.5556y^2$$

$$= 100y + 5555.555556y - 555555.5556y^2$$

$$= 5655.555556y - 555555.5556y^2$$

$$= y (5655.555556 - 555555.5556y)$$

Hence the velocity distribution:

$$u = y (5655.555556 - 555555.5556y)$$

② Discharge per unit width of q :

$$q = \int_0^b u dy$$

$$= \int_0^{0.01} (5655.555556y - 555555.5556y^2) dy$$

Velocity & Shear stress distribution between the plates

$$h_1 - h_2 = \left(\frac{Z_1 + Z_2}{\omega} \right) - \left(\frac{Z_1 + Z_2}{\omega} \right) + 0$$

$$= \left(\frac{250 \times 10^3 + 1}{1.26 \times 9810} \right) - \left(\frac{80 \times 10^3}{1.26 \times 9810} \right) + 0$$

$$= 21.22555539 - 6.472179726$$

$$= 14.75337766$$

$$h_1 - h_2 \approx 14.755 \approx \sqrt{2}m \text{ or } \underline{1.414m}$$

$$\frac{\partial h}{\partial x} = - \frac{14.755}{1.414} \approx -10.435$$

$$\frac{\partial p}{\partial x} = \omega \frac{\partial h}{\partial x} = (1.26 \times 9810) \times (-10.435)$$

$$= -128983 \text{ N/m}^3 \text{ approx.}$$

$$u = \frac{u}{b} \cdot y - \frac{1}{2\mu} \left(\frac{\partial p}{\partial x} \right) (by - y^2)$$

$$u = - \frac{1.5}{0.01} y - \frac{1}{2 \times 0.1} \times (-128.983 \times 10^3) (0.01y - y^2)$$

$$= -150y + 716.5722222 - 71657.22222y^2$$

$$u = 566.57y - 71657y^2$$

Velocity distribution is

$$u = 566.57y - 71657 \times 10^4 y^2$$

Shear distribution

$$\tau = \mu \cdot \frac{u}{b} - \frac{1}{2} \frac{\partial p}{\partial x} (b - 2y)$$

$$\begin{aligned}
 & \left[5655.555556 \times \frac{y^2}{2} - 555555.5556 \times \frac{y^3}{3} \right]_0^{0.01} \\
 & = \left[5655.555556 \times \frac{0.01^2}{2} - 555555.5556 \times \frac{0.01^3}{3} \right] \\
 & = (0.2827777778 - 0.1851851852) \\
 & = 0.0975925926 \text{ m}^3/\text{s}
 \end{aligned}$$

① The shear stress on the upper plate.

$$\text{Shear stress} = \mu \left(\frac{\partial u}{\partial y} \right)$$

$$= \mu \frac{\partial}{\partial y} (5655.555556y - 555555.5556y^2)$$

$$\tau = 0.0009 (5655.555556 - 111111.111y)$$

$$y = b = 0.01 \text{ m} \quad \text{from top plate}$$

$$\tau_0 = 0.0009 (5655.555556 - 111111.111 \times 0.01)$$

$$= -4.909999999$$

$$= -5 \text{ N/m}^2$$

② Viscosity = 0.9 Ns/m^2

$$\text{Density} = 1260 \text{ kg/m}^3 = 1.26$$

$$\text{Distance between plates} = 10 \text{ mm} = 0.01 \text{ m}$$

$$\text{Velocity of the upper plate} = 1.5 \text{ m/s}$$

$$p_1 = 250 \text{ kN/m}^2$$

$$p_2 = 80 \text{ kN/m}^2$$

$$\tau = 0.9 \times \left(\frac{-1.5}{0.01} \right) - \frac{1}{2} \times (-128983)(0.01 - 2y)$$

$$= 135 + 644.92 - 128983y$$

$$\tau = 509.92 - 128983y$$

Maximum flow velocity u_{max}

$$\frac{du}{dy} = 0$$

$$\frac{d}{dy} (566.57y - 71657y^2) = 0$$

$$566.57 - 143314y = 0$$

$$y = 3.9533 \times 10^{-3} \text{ m}$$

$$\begin{aligned} V_{max} &= 566.57 (3.95 \times 10^{-3}) - 71657 (3.95 \times 10^{-3})^2 \\ &= 2.238 - 1.118 \\ &= 1.12 \text{ m/s} \end{aligned}$$

The shear stress on the upper plate

$$\begin{aligned} \tau_{0.01} &= 509.92 - 128983 \times 0.01 \\ &= -779.91 \text{ Pa} \approx -780 \text{ N/m}^2 \end{aligned}$$