1) Maxwell's equation in integral form

1. $\oiint_S D \cdot ds = Q_{enc}$ = Amount of charge within surface

2. $\oiint_S B \cdot ds = 0$ — Gauss law (magnetism)

3. $\oint_L E \cdot dl = -\iint_S \frac{\partial B}{\partial t} \cdot ds$ — Faraday's law

4. $\oint_L H \cdot dl = I_{enc} + \iint_S \frac{\partial D}{\partial t} \cdot ds$

$\int (\nabla \times A) \cdot ds = \oint A \cdot dl$ [Stokes law]

Maxwell's equation in Differential form

$\vec{\nabla} \cdot D = \rho$

$\vec{\nabla} \cdot \vec{B} = 0$ — Gauss law

$\vec{\nabla} \times E = -\frac{\partial \vec{B}}{\partial t}$ — Faraday's law of magnetism

$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$ — Ampere's law

* $\vec{\nabla} \cdot \vec{B} = 0$ — Gauss law of magnetism is derived from Gauss law
* $\oiint \vec{B} \cdot ds = 0$ — Integral form of Gauss law of magnetism is derived from Gauss
* $\oint E \cdot dl = -\frac{d}{dt} \int \vec{B} \cdot ds$ — integral form of ~~Gauss~~ Faraday's law of magnetism
* $\vec{\nabla} \times E = -\frac{\partial E}{\partial t}$ — Differential form of Faraday's law of magnetism
* $(\nabla \times H) = \vec{J} + \frac{\partial D}{\partial t}$ — Differential form in Ampere

b) Maxwell found that [illegible] in Ampere's law and he modified Ampere's law to include time varying electric fields. For ~~example~~ Ampere's circuital law to be consistent Maxwell assumed that there has to be some current existing between the plates of the capacitor. Ampere law in it's integral form [illegible]

$I = \oint \vec{H} \cdot dl$

→ Maxwell introduced a term called displacement current density $\frac{\partial D}{\partial t}$

Recall $V = Ed$

$i_c = C\frac{dV}{dt} = \frac{\epsilon A}{d} \cdot d\left(\frac{dE}{dt}\right)$

δt

1b Maxwell found the shortcoming in ampere's law and he modified ampere's law to include time-varying electric fields. For Ampere's circuital law to be, maxwell ~~[illegible]~~ to ~~be~~ assumption make the set of equations mathematically consistent for non static fields, without changing the

$$\frac{\delta^2 E_y}{\delta x^2} = \gamma^2 E_y$$

where,

H = Magnetic field strength (A/m)

D = electric flux density, (C/m²)

$(\partial D/\partial t)$ = displacement electric current density (A/m²)

J = Conduction current density (A/m²)

E = electric field (V/m)

B = magnetic flux density wb/m² or Tesla

$(\partial B/\partial t)$ = time-derivative of magnetic fluxdensity (wb/m²s)

B is called magnetic current density (C/m³)

ρ_v = Volume charge density (C/m³)

1a Maxwell's Equations in their Integral forms

$\int D.ds = Q$ —> Gauss' law (electric)

$\int B.ds = 0$ —> Gauss' law (magnetic)

$\oint E.dl = \int \frac{dB}{dt}.ds = V$ —> Faraday

$I = \oint \vec{H}.dl = \int J + \frac{d\bar{D}}{dt} \delta s$ —> Ampere

Maxwell's equations in their differential form

$\bar{V}.D = \rho_v$ —> derived from Gauss' law (electric)

$\bar{V}.\bar{B} = 0$ —> derived from Gauss' law (magnetic)

$\vec{V} \times E = -\frac{\delta \vec{B}}{\delta t}$ —> derived from faraday

$\vec{V} \times \vec{h} = J + \frac{\delta \bar{D}}{\delta t}$ —> derived from Ampere

2biii For characteristic impedance of the medium

$v = c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$ → (In free space or vacuum)

$v = c = \frac{1}{\sqrt{\mu \varepsilon}}$ = (in any other non-conducting medium)

$v = \frac{1}{\sqrt{\mu \varepsilon}} = \frac{1}{\sqrt{\mu_r \mu_0 \varepsilon_r \varepsilon_0}}$

$= \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \times \frac{1}{\sqrt{\mu_r \varepsilon_r}} = \frac{c}{\sqrt{\mu_r \varepsilon_r}}$

$\therefore c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$

where $Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0} \cdot \frac{\varepsilon_0}{H_0}}$ → [characteristic impedance of the medium]

2ci $v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$

where $\mu_0 = 4\pi \times 10^{-7}$ N/m

$\varepsilon_0 = 8.854 \times 10^{-12}$ F/M

$\therefore v = \frac{1}{\sqrt{4\pi \times 10^{-7} \times 8.854 \times 10^{-12}}} = 3 \times 10^{8}\ ms^{-1}$

xiii $\cancel{Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}}$ $Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}$

Where $\mu_0 = 4\pi \times 10^{-7}\ Hm^{-1}$

$\varepsilon_0 = \frac{1}{36\pi} \times 10^{-9}\ fm^{-1}$

$$Z_0 = \sqrt{\frac{4\pi \times 10^{-7}}{\frac{1}{36\pi} \times 10^{-9}}}$$

$$= \sqrt{4\pi (36\pi) 10^{2}}$$

$$= \sqrt{144\pi \times 10^{2}}$$

$$= 120\pi$$

$Z_0 = 120\pi$

$Z_0 = 371\,\Omega$

Wave propagation in a conducting medium

From Ampere's & Faraday's law

laws of Ampere and Gauss for static fields -

However, as a consequence, it predicts that a magnetic field induces an electric field and vice versa. Therefore these equations allow self-sustaining "electromagnetic waves" to travel through empty space.

2b

$$\frac{\partial H_z}{\partial x} = -\varepsilon_0 \frac{\partial E_y}{\partial t} \quad \ldots (1)$$

$$\frac{\partial E_y}{\partial x} = -\mu_0 \frac{\partial H_z}{\partial t} \quad \ldots (2)$$

Eqns 1 & 2 are coupled equations at the moments

Decoupling them ~~eth~~ we have;

$$\frac{1}{dt}\left(\frac{dH_z}{dx}\right) = -\varepsilon_0 \frac{\delta^2 E_y}{\delta t^2}$$

or $\frac{d^2 H_z}{dt^2 dx} = -\varepsilon_0 \frac{\delta^2 E_y}{\delta t^2}$ ... * (2nd differential)

$$\frac{d^2 E_y}{dx^2} = -\mu_0 \frac{d^2 H_z}{dx\,dt} \quad \text{-* **(2nd differential) *}$$

Putting * in ** we have,

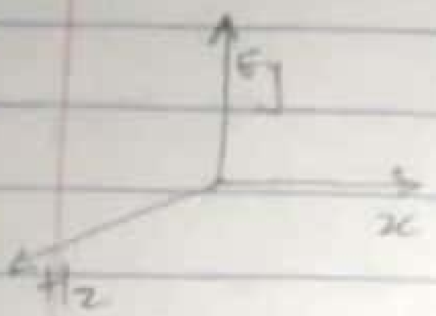
$$\frac{d^2 E_y}{dx^2} = -\mu_0 \left(-\varepsilon_0 \frac{d^2 E_y}{dt^2}\right)$$

$$= \mu_0 \varepsilon_0 \frac{\delta^2 E_y}{\delta t^2}$$

$$\therefore \frac{d^2 E_y}{dx^2} = \mu_0 \varepsilon_0 \frac{d^2 E_y}{dt^2} \longrightarrow \{\text{Wave equation}\}$$

2(a) For E_y

We have to force this assumption of saying that only E_y and H_z exist, and they only vary in the x-direction based on H_z



$$\vec{\nabla} \times \vec{H} = \begin{vmatrix} +\hat{x} & -\hat{y} & +\hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ H_x & H_y & H_z \end{vmatrix}$$

$$+\hat{x}\left(\cancel{\frac{dH_z}{dy}} - \cancel{\frac{dH_y}{dz}}\right) - \hat{y}\left(\frac{dH_z}{dx} - \cancel{\frac{dH_x}{dz}}\right) + \hat{z}\left(\cancel{\frac{dH_y}{dx}} - \cancel{\frac{dH_x}{dy}}\right)$$

$$\therefore -\hat{y}\left(\frac{dH_z}{dx}\right)$$

$$\hat{y}\frac{dH_z}{dx} = \hat{y}\cdot\frac{\partial D_y}{\partial t}$$

~~from~~ $\cancel{\frac{\partial E_y}{dx}} = \cancel{-\frac{\partial B}{\partial t}}$

$$= -\varepsilon_0\frac{\partial E_y}{\partial t} \quad (\text{from } -D = \varepsilon_0 E)$$

$$\therefore \frac{dH_z}{dx} = -\varepsilon_0\frac{\partial E_y}{\partial t}$$

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2bii) we derive the expression for wave velocity from the wave equation in both E_y & H_z which is;

$$\frac{d^2E_y}{dx^2} = \mu_0 \varepsilon_0 \frac{d^2 E_y}{dt^2} \quad \cdots (1)$$

So we have;

$$\cancel{\frac{d^2 E_y}{dt^2} = \frac{1}{\mu_0 \varepsilon_0} \frac{d^2 E_y}{dx^2}}$$

$$\therefore \frac{d^2 E_y}{dx^2} = -\mu_0 \frac{\partial}{\partial x}\left(\frac{\partial h_z}{\partial t}\right) \quad \cdots (2)$$

Equating eqn (1) & (2) we have;

$$\frac{\partial}{\partial t}\left(\frac{\partial h_z}{\partial x}\right) = -\varepsilon_0 \frac{\partial^2 E_y}{\partial t^2}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial h_z}{\partial t}\right) = \frac{-1}{\mu_0} \frac{\partial^2 E_y}{\partial t^2}$$

$$-\varepsilon_0 \frac{d^2 E_y}{\partial t^2} = \frac{-1}{\mu_0} \frac{\partial^2 E_y}{\partial x^2}$$

$$\frac{\partial^2 E_y}{\partial t^2} = \frac{1}{\mu_0 \varepsilon_0} \frac{\partial^2 E_y}{\partial x^2}$$

$$\therefore V = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

for the, assuming only E_y & H_z exist, and they only vary or polarize in the y-direction based on E_y

$$\vec{\nabla} \times \vec{E} = \begin{vmatrix} x & y & z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ E_x & E_y & E_z \end{vmatrix}$$

$$= \hat{x}\left(\frac{\cancel{\partial E_z}}{\partial y} - \frac{\cancel{\partial E_y}}{\partial z}\right) - y\left(\frac{\cancel{\partial E_z}}{\partial x} - \frac{\cancel{\partial E_x}}{\partial z}\right) + \hat{z}\left(\frac{\partial E_y}{\partial x} - \frac{\cancel{\partial E_x}}{\partial y}\right)$$

$$\therefore \hat{z}\left(\frac{\partial E_y}{\partial x}\right)$$

$$= \left(\frac{\partial E_y}{\partial x}\right) = -z\frac{\partial B}{\partial t}, \quad -\mu_0\frac{\partial H_z}{\partial t} = \frac{\partial E_y}{\partial x} \text{ from } B = \mu_0 H$$

$$\frac{\partial E_y}{\partial x} = -\mu_0\frac{\partial H_z}{\partial t}$$