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19/ENG04/010
Elect / Elect
MAT 104 → Calculus

① Hint $\frac{(11-3x)}{x^2+2x-3}$

Solution

$$\int \frac{11-3x}{x^2+2x-3}\,dx = \int \frac{11-3x}{(x+3)(x-1)}\,dx = \int \frac{A}{x+3} + \frac{B}{x-1}\,dx$$

Without integrals

$$11-3x = A(x-1)+B(x+3)$$

when $x=1$

$$11-3(1) = A(1-1)+B(1+3)$$

$$11-3 = 0+4B$$

$$8 = 4B$$

$$B = 2$$

When $x=-3$,

$$11-3(-3) = A(-3-1)+B(-3+3)$$

$$11+9 = -4A+0$$

$$20 = -4A$$

$$A = -5$$

$$\therefore \int \frac{11-5x}{(x+3)(x-1)}\,dx = \int \frac{-5}{x+3}\,dx + \int \frac{2}{x-1}\,dx$$

let $u = x+3$ | let $u = x-1$

$\frac{du}{dx} = 1$ | $\frac{du}{dx} = 1$

$du = dx$ | $du = dx$

$\Rightarrow \int \frac{-5}{u}\,du$ | $\Rightarrow \int \frac{2}{u}\,du$

$= -5(\ln u)$ | $= 2(\ln u)$

$= -5\ln u = -5\ln(x+3)$ | $= 2\ln u = 2\ln(x-1)$

$$\therefore \int \frac{11-3}{x^2+2x-3}\,dx = -5\ln(x+3) + 2\ln(x-1) + C$$

2 $\frac{2x^2-4x-35}{(x+1)(x-2)(x+3)}$

solution

$$\frac{2x^2-4x-35}{(x+1)(x-2)(x+3)} = \frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{x+3}$$

L.C.M

$$2x^2-4x-35 = A(x-2)(x+3) + B(x+1)(x+3) + C(x+3)(x-2)$$

When $x = 2$

$$2(2)^2-4(2)-35 = A(2-2)(2+3) + B(2+1)(2+3) + C(2+1)(2-2)$$

$$8-18-35 = 0 + B(3\times 5) + 0$$

$$-45 = 15B \quad \therefore B = -3$$

When $x = -3$,

$2(-3)^2 - 4(-3) - 35 = A(-3-1)(-3+3) + B(1-3)(-3+3) + C(-3+1)(-5-2)$

$18 + 12 - 35 = 0 + 0 + C(-2)(-5)$

$10 = 10C$

$C = 1$

When $x = -1$

$2(-1)^2 - 4(-1) - 35 = A(-1-1)(-1+3) + B(-1+1)(-1+3) + C(-1+1)(-1-2)$

$2 + 4 - 35 = A(-3)(2) + 0 + 0$

$-24 = -6A$

$A = 4$

$$\int \frac{2x^2 - 4x - 35}{(x+1)(x-2)(x+3)}\,dx = \int \frac{4}{x+1}\,dx + \int \frac{-3}{x-2}\,dx + \int \frac{1}{x+3}\,dx$$

Let $u = x+1$ Let $u = x-2$ Let $u = x+3$

$\frac{du}{dx} = 1$ $\frac{du}{dx} = 1$ $\frac{du}{dx} = 1$

$du = dx$ $du = dx$ $du = dx$

$= \int \frac{4}{u}\,du$ $= \int \frac{-3}{u}\,du$ $= \int \frac{1}{u}\,du$

$= 4 \ln u$ $= -3 \ln u$ $= \ln u$

$$\therefore \int \frac{2x^2 - 4x - 35}{(x+1)(x-2)(x+3)}\,dx = 4\ln(x+1) - 3\ln(x-2) + \ln(x+3) + c$$

3 $\frac{1}{x^2+121}$

Solution

$$\int \frac{1}{x^2+121}\,dx = \int \frac{dx}{x^2+121} = \int \frac{dx}{x^2+11^2}$$

$\therefore$ Let $x = 11\tan\theta$ $\quad\therefore \theta = \tan^{-1}(x/11)$

$$\frac{dx}{d\theta} = 11\sec^2\theta$$

$$dx = 11\sec^2\theta\, d\theta$$

$$x^2 + 11^2 = 11^2\tan^2\theta + 11^3$$

$$= 11^2(\tan^2\theta + 1)$$

$$= 11^2\sec^2\theta = 121\sec^2\theta$$

$$= \int \frac{11\sec^2\theta\, d\theta}{121\sec^2\theta}$$

$$= \int \frac{d\theta}{11} = \frac{1}{11}\int d\theta$$

$$= \frac{1}{11}(\theta) + C$$

$$\int \frac{1}{x^2+121}\,dx = \frac{1}{11}\tan^{-1}(x/11) = \frac{\tan^{-1}(x/11)}{11} + C$$