

AFINIMI JOHN
Computer Engineering

19/ENG02/028

Serial NO :- 50

MAT 104

Assignment

Integrate the following with respect to their variable

1 $\frac{(11-3x)}{x^2+2x-3}$

$$\frac{(11-3x)}{x^2-2x-3} = \frac{11-3x}{(x-1)(x+3)}$$

$$\frac{11-3x}{(x-1)(x+3)} = \frac{A}{x-1} + \frac{B}{x+3}$$

$$\frac{11-3x}{(x-1)(x+3)} = \frac{A(x+3) + B(x-1)}{(x-1)(x+3)}$$

Multiply both sides by $(x-1)(x+3)$

$$11-3x = A(x+3) + B(x-1)$$

At $x = -3$, we have

$$B(-4) = 11 - 3(-3)$$

$$-4B = 11 + 9$$

$$-4B = 20$$

$$B = -\frac{20}{4}$$

$$B = -5$$

At $x = 1$; $A(4) = 11 - 3(1)$

$$4A = 11 - 3$$

$$4A = 8$$

$$A = \frac{8}{4}$$

$$A = 2$$

$$\therefore \int \frac{2}{x-1} dx + \int \frac{-5}{x+3} dx = \int \frac{11-3x}{x^2+2x-3} dx$$

$$\int \frac{2 dx}{x-1} + \int \frac{-5 dx}{x+3} = \int \frac{11-3x}{x^2+2x-3} dx$$

$$2 \int \frac{dx}{x-1} - 5 \int \frac{dx}{x+3}$$

$$u = x-1$$

$$u = x+3$$

$$\frac{du}{dx} = 1$$

$$\frac{du}{dx} = 1$$

$$du = dx$$

$$du = dx$$

$$2 \int \frac{du}{u}$$

$$-5 \int \frac{du}{u}$$

$$2 \ln u$$

$$-5 \ln(x+3)$$

$$= 2 \ln(x-1)$$

$$= 2 \ln(x-1) - 5 \ln(x+3)$$

∴

$$\text{Then } \int \frac{11-3x}{x^2+2x-3} = 2 \ln(x-1) - 5 \ln(x+3)$$

$$2. \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)}$$

$$\int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} dx = \frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{x+3}$$

$$\frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} = \frac{A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x-2)}{(x+1)(x-2)(x+3)}$$

$$2x^2 - 9x - 35 = Ax^2 + x - 6 + B(x^2 + 4x + 3) + C(x^2 - x - 2)$$

$$2x^2 - 9x - 35 = Ax^2 + Ax - 6A + Bx^2 + 4Bx + 3B + Cx^2 - Cx - 2C$$

$$2x^2 - 9x - 35 = (A+B+C)x^2 + (A+4B-C)x + (-6A+3B-2C)$$

$$A+B+C = 2 \quad \text{--- i}$$

$$A+4B-C = -9 \quad \text{--- ii}$$

$$-6A+3B-2C = -35 \quad \text{--- iii}$$

$$\text{From i } A = 2 - B - C \quad \text{--- iv}$$

Put eqn iv into ii and iii

$$(2 - B - C) + 4B - C = -9$$

$$4B - B - C - C = -9 - 2$$

$$3B - 2C = -11$$

$$3B - 2C = -11 \times 3$$

$$-6(2 - B - C) + 3B - 2C = -35$$

$$12B + 3B + 6C - 2C = -35 + 12$$

$$15B + 4C = -23$$

$$3B - 2C = -11 \times 3$$

$$15B + 4C = -23 \times 1$$

$$15B - 6B = -33$$

$$\underline{-15B + 4C = -23}$$

$$-10C = -10$$

$$C = \frac{-10}{-10}$$

$$C = 1$$

$$3B - 2 = -11$$

$$3B = -11 + 2$$

$$3B = -9$$

$$B = \frac{-9}{3}$$

$$B = -3$$

$$A = 2 - (-3) - 1$$

$$A = 2 + 3 - 1 = 5 - 1$$

$$A = 4$$

$$\frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} = \int \frac{4dx}{x+1} + \int \frac{-3dx}{x-2} + \int \frac{e dx}{x+1}$$

$$\frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} = 4 \int \frac{e dx}{x+1} - 3 \int \frac{e dx}{x-2} + \int \frac{e dx}{x+1}$$

$$\frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} = \begin{matrix} u = x+1 & u = x-2 & u = x+3 \\ \frac{du}{dx} = 1 & \frac{du}{dx} = 1 & \frac{du}{dx} = 1 \\ dx = du & dx = du & dx = du \end{matrix}$$

$$\int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} = 4 \int \frac{du}{u} - 3 \int \frac{du}{u} + \int \frac{du}{u}$$

$$= 4 \ln u - 3 \ln u + \ln u$$

$$\int \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} = 4 \ln(x+1) - 3 \ln(x-2) + \ln(x+3)$$

$$3 \int \frac{1}{x^2 + 121} dx = \int \frac{dx}{x^2 + 121}$$

$$= \int \frac{dx}{x^2 + 11^2}$$

$$x = 11 \tan \theta$$

$$\frac{dx}{d\theta} = 11 \sec^2 \theta$$

$$dx = 11 \sec^2 \theta d\theta$$

$$x^2 + 11^2 = 11^2 \tan^2 \theta + 11^2 = 11^2 (\tan^2 \theta + 1)$$

$$\text{However, } 1 + \tan^2 \theta = \sec^2 \theta$$

$$\int \frac{11 \sec^2 \theta d\theta}{121 \sec^2 \theta} = \int \frac{d\theta}{11} = \frac{1}{11} \int d\theta$$

$$\frac{1}{11} (\theta) + C$$

$$\text{and } x = 11 \tan \theta$$

$$\tan \theta = \frac{x}{11}$$

$$\theta = \tan^{-1} \frac{x}{11}$$

$$\int \frac{1}{x^2 + 121} dx = \frac{1}{11} \tan^{-1} \frac{x}{11} + C$$