

NAME: MBIA, MALORY ANNE
DEPARTMENT: INDUSTRIAL CHEMISTRY
COURSE: MAT 104
COLLEGE: SCIENCES
Matric Num: 19/5409/004

Number 11.

$$\lim_{x \rightarrow 0} \frac{(1 - \cos x)}{x}$$

$$\lim \left(\frac{1 - \cos x}{x} \right) \frac{(1 + \cos x)}{(1 + \cos x)}$$

$$\lim_{x \rightarrow 0} \frac{(1 - \cos^2 x)}{x(1 + \cos x)}$$

Recall the identity, $\sin^2 x + \cos^2 x = 1$
therefore $1 - \cos^2 x = \sin^2 x$

$$\lim_{x \rightarrow 0} \frac{(1 - \cos^2 x)}{x(1 + \cos x)}$$

$$\lim_{x \rightarrow 0} \frac{\sin^2 x}{x(1 + \cos x)} = \frac{(\sin x)(\sin x)}{x(1 + \cos x)}$$

$$\lim_{x \rightarrow 0} \frac{\sin^2 x}{x(1 + \cos x)} = \frac{(\sin x)(\sin x)}{x(1 + \cos x)}$$

$$\lim = \frac{\sin x}{x} \frac{\sin x}{(1 + \cos x)}$$

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} \quad \lim_{x \rightarrow 0} \frac{\sin x}{1 + \cos x}$$

Recall

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$\lim_{x \rightarrow 0} \frac{\sin x}{1 + \cos x}$$

$$= \frac{\sin 0}{1 + \cos 0}$$

$$= \frac{0}{1+1} \Rightarrow \frac{0}{2} \Rightarrow \frac{0}{2}$$

Number 2:

$$y = -3 \quad u = -4 \tan 7x \quad w = e^{3x}$$

$$\frac{dy}{dx} = 0 \quad \frac{du}{dx} = 7 \sec^2 7x \quad \frac{dw}{dx} = 3e^{3x}$$

$$\frac{dy}{dx} = y^{20} \left[\frac{1}{3} (0) + \frac{1}{\tan 7x} (7 \sec^2 7x) + \frac{1}{e^{3x}} (3e^{3x}) \right]$$

$$y \left[0 + \frac{7 \sec^2 7x}{\tan 7x} + 3 \right]$$

$$\frac{dy}{dx} = \left[\frac{7 \sec^2 7x + 3}{\tan 7x} \right] \left[-3 \tan 7x e^{3x} \right]$$

Number 3:

$$y = \cos 3x$$

$$y = \Delta y = \cos(3x + \Delta x)$$

$$\Delta y = \cos(3x + \Delta x) - \cos 3x$$

$$\text{Recall } \cos A - \cos B = -2 \sin \frac{A+B}{2} = \frac{\sin A - B}{2}$$

Comparing equation $B = 3x$

$$\frac{A+B}{2} = \frac{3x + \Delta x + 3x}{2} = \frac{6x + \Delta x}{2}$$

and

$$\frac{A-B}{2} = \frac{3x + \Delta x - 3x}{2} = \frac{\Delta x}{2}$$

$$\text{Hence, } \Delta y = \cos(3x + \Delta x) - \cos 3x \\ = -2 \sin \left(\frac{6x + \Delta x}{2} \right) \sin \frac{\Delta x}{2}$$

\therefore This value determines the number is divided by $\frac{1}{2} + \frac{1}{2}$

$$\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \frac{\sin 0}{0} = 1$$

$$\frac{\Delta y}{\Delta x} = -2 \sin \left(\frac{6x + \Delta x}{2} \right) \frac{\left(\sin \frac{\Delta x}{2} \right) \times \frac{1}{2}}{3x \times \frac{1}{2}}$$

$$\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = -2 \sin(6x + 0) \lim_{\Delta x \rightarrow 0} \frac{\sin \left(\frac{\Delta x}{2} \right)}{\frac{\Delta x}{2}}$$

$$\text{since } = \lim_{\Delta x \rightarrow 0} \frac{\sin(\Delta x)}{\frac{\Delta x}{2}} = 1$$

$$\text{Hence } \frac{dy}{dx} = -2 \sin 6x.$$

Number 4:

$$f(x) = 2x^3 - 7x \text{ and } g(x) = -3x$$

$$(f-g)(5) = f(5) - g(5)$$

$$(f-g)(5) = [2(5)^3 - 7(5)] - [-3(5)]$$

$$(f-g)(5) = 2(125) - 35 - [-15]$$

$$(f-g)(5) = 250 - 35 + 15$$

$$(f-g)(5) = 230$$

Number 5:

$$f(x) = 4x^2 + 2 \text{ and } g(x) = 2x + 3$$

$$(fg)(x) = (4x^2 + 2)(2x + 3) + 2$$

$$= 4(2x + 3)(2x + 3) + 2$$

$$= 4(4x^2 + 6x + 6x + 9) + 2$$

$$= 4(4x^2 + 12x + 9) + 2$$

$$= 16x^2 + 48x + 36 + 2$$

$$= 16x^2 + 48x + 38$$

Number 6:

$$2x^2 + 2xy + y^2 = 1020$$

$$2x + 2x \frac{dy}{dx} + 2y + 2y \frac{dy}{dx} = 0$$

$$2x \frac{dy}{dx} + 2y \frac{dy}{dx} = -2x - 2y$$

$$\frac{dy}{dx} (2x + 2y) = -2x - 2y$$

$$\frac{dy}{dx} = \frac{-2x - 2y}{2x + 2y}$$

Number 7

$$y = x^2 \cos x$$

$$u = x^2 \quad y = \cos x$$

$$\frac{du}{dx} = 2x$$

$$v = \cos x$$

$$\frac{dv}{dx} = -\sin x$$

$$dy = u \frac{du}{dx} + v \frac{dv}{dx}$$

$$= (x^2) (-\sin x) + (2x) (\cos x)$$

$$= -x^2 \sin x + 2x \cos x$$

$$= x(-x \sin x + 2 \cos x)$$

$$x(2 \cos x - x \sin x) \text{ or } (2x \cos x - x^2 \sin x)$$