Mat 104 MBBS

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1) $\int \frac{2x}{\sqrt{4x^2-1}} \, dx$

$u = 4x^2 - 1$

$x = \sqrt{\frac{u+1}{4}}$; $x = \frac{(u+1)^{1/2}}{2}$

$\frac{dx}{du} = \frac{1}{2}(u+1)^{-1/2} \cdot \frac{1}{2}$

$\frac{dx}{du} = \frac{1}{4}(u+1)^{-1/2}$

$dx = du / 4(u+1)^{1/2}$

$\int \frac{2x}{\sqrt{4x^2-1}} = \int \frac{2x}{u^{1/2}} \cdot \frac{du}{4(u+1)^{1/2}}$

$\int \frac{2x}{\sqrt{4x^2-1}} = \frac{1}{2} \int \frac{u}{u^{1/2}} \cdot \frac{du}{(u+1)^{1/2}}$

$\int \frac{2x}{\sqrt{4x^2-1}} = \frac{1}{4} \int \frac{(u+1)^{1/2}}{u^{1/2}} \cdot \frac{du}{(u+1)^{1/2}}$

$\int \frac{2x}{\sqrt{4x^2-1}} = \frac{1}{4} \int u^{-1/2} \cdot du$

$\int \frac{2x}{\sqrt{4x^2-1}} = \frac{1}{4} \left[\frac{u^{1/2}}{1/2} \right] + C$

$$\int \frac{2x}{\sqrt{4x^2-1}} = \frac{1}{4}\left[2u^{1/2}\right] + c$$

$$= \frac{1}{4^2} \cdot 2(4x^2-1)^{1/2} + c$$

$$\int \frac{2x}{\sqrt{4x^2-1}} \cdot dx = \frac{\sqrt{(4x^2-1)}}{2} + c$$

$$\text{or } \frac{1}{2}\sqrt{4x^2-1} + c$$

2) $\displaystyle\int \frac{\sin^{-1} x}{\sqrt{1-x^2}} \cdot dx$

Solution

$U = 1 - x^2$, $x = \sqrt{u-1}$

$\frac{du}{dx} = -2x$, $dx = -\frac{du}{2x}$

$$\int \frac{\sin^{-1} x}{\sqrt{1-x^2}} = \int \frac{\sin^{-1} x}{\sqrt{u}} \cdot \frac{du}{2x}$$

$$\int \frac{\sin^{-1} x}{(\sqrt{1-x^2})} = \frac{1}{2}\int \sin^{-1}(\sqrt{u-1})$$

$$\int \frac{\sin^{-1} x}{\sqrt{1-x^2}} = \frac{1}{2}\int \frac{\sin^{-1} x}{(1-x^2)^{1/2}} \cdot \frac{du}{x}$$

$$\int \frac{\sin^{-1} x}{\sqrt{1-x^2}} = \frac{1}{2}\int \frac{\sin^{-1} x}{\sin^{-1} x} \cdot \frac{du}{x})$$

$$\int \frac{\sin^{-1} x}{\sqrt{1-x^2}} = \frac{1}{2} \sin^{-1} x \cdot \sin^{-1} x + c$$

$$\therefore \int \frac{\sin^{-1} x}{\sqrt{1-x^2}} = \frac{(\sin^{-1} x)^2}{2} + c$$

3) $\int (\tan x)^6 \sec^2 x \, dx$

Solution

$u = \tan x$

$du/dx = \sec^2 x$

$dx = du/\sec^2 x$

$\int (\tan x)^6 \sec^2 x = \int u^6 \sec^2 x \cdot du/\sec^2 x$

$\int (\tan x)^6 \sec^2 x = u^7/7 + c$

$\int (\tan x)^6 \sec^2 x = \frac{\tan x^7}{7} + C$