

Name: FSE - FJAKPOVI NYOYOGHANEMatric no: 19/MHS01/159course code: MAT 104 (1st Integration - Assignment).
100 Level.

$$(1) \int \frac{2x}{\sqrt{4x^2-1}} \cdot dx$$

Solution:

$$u = \sqrt{4x^2-1}$$

$$u^2 = 4x^2 - 1$$

$$x = \sqrt{\frac{u^2+1}{4}}$$

$$x = \frac{(u^2+1)^{1/2}}{2}$$

OR

$$u = \sqrt{4x^2-1}$$

$$x = \sqrt{\frac{u+1}{4}}$$

$$x = \frac{(u+1)^{1/2}}{2}$$

$$\frac{dx}{du} = \frac{1}{2} (u+1)^{-1/2} \cdot \frac{1}{2}$$

$$\frac{dx}{du} = \frac{1}{4} (u+1)^{-1/2}$$

$$\frac{dx}{du} = \frac{du}{4(u+1)^{1/2}}$$

Continuation of no. 1:

$$= \int \frac{2x}{u^{1/2}} \cdot \frac{du}{4(u+1)^{1/2}}$$

$$= \frac{1}{2} \int \frac{x}{u^{1/2}} \cdot \frac{du}{(u+1)^{1/2}}$$

$$= \frac{1}{4} \int \frac{(u+1)^{1/2}}{u^{1/2} \cdot (u+1)^{1/2}} \cdot du$$

$$= \frac{1}{4} \int u^{-1/2} \cdot du$$

$$= \frac{1}{4} \left[\frac{u^{1/2}}{1/2} \right] + C$$

$$= \frac{1}{4} \left[2u^{1/2} \right] + C$$

$$= \frac{1}{4 \cdot 2} \cdot \sqrt{(4x^2 - 1)} + C$$

$$\therefore \int \frac{2x}{\sqrt{4x^2 - 1}} \cdot dx = \frac{\sqrt{(4x^2 - 1)}}{2} + C$$

$$\text{OR } \frac{1}{2} \sqrt{4x^2 - 1} + C$$

$$\textcircled{2} \int \frac{\sin^{-1} x}{\sqrt{1-x^2}} \cdot dx$$

Solution

$$u = \sin^{-1} x$$

$$\frac{du}{dx} = \frac{1}{\sqrt{1-x^2}}$$

$$dx = \sqrt{1-x^2} du$$

$$= \int \frac{u}{\sqrt{1-x^2}} \cdot \sqrt{1-x^2} du$$

$$= \int \sin^{-1} x du$$

$$= \frac{(\sin^{-1} x)^2}{2} + C$$

$$\therefore \int \frac{\sin^{-1} x}{\sqrt{1-x^2}} \cdot dx = \frac{(\sin^{-1} x)^2}{2} + C$$

$$(3) \int (\tan x)^6 \sec^2 x \, dx$$

Solution:

$$u = \tan x$$

$$\frac{du}{dx} = \sec^2 x$$

$$dx = \frac{du}{\sec^2 x}$$

$$= \int u^6 \sec^2 x \cdot \frac{du}{\sec^2 x}$$

$$= \frac{u^7}{7} + C$$

$$\therefore \int (\tan x)^6 \sec^2 x \, dx = \frac{(\tan x)^7}{7} + C$$