(1) Solution.

When considering waiting time we have to focus on two thing the custumers find the service rendered to the customer.

For example;: A line customer at the bank trying to withdraw money from a bank clerk.

From the above we ~~sat~~ say a Queue is formed and a service is rendered the system that performs the service to the customer is called a service channel or service faculty. The reason why we study waiting line theory is to understand the fluntuation of service demand and help improve the ~~serv~~ service of a system.

(2) Explain with suitable examples about poisson arrival pattern and exponential Service pattern.

Ans.

Poisson Arrival pattern:- Arrival pattern of customers at a bank, this is an example of Poisson arrival pattern or poisson distribution. Poisson distribution corresponds to completely random arrival and it is assumed that arrival are completely independent of other arrivals as well as any condition of the waiting time.

Exponential Service pattern: A mechanic working on 4 cars and it takes him 8 hours to work on each, it forms a exponential distribution

This describes the service time as the probabili
that a particular service time will be less than
or equal to a given amount of time.

(17) Solution.

Date, finite Population, λ = Arrival rate $(1/5)$ $= 0.2$, μ = Service rate $= \mu = (1/1) = 1$

Probability of the empty system $= P_0 =$

$P_0 = 1/\sum_{n=0}^{4} [4!/(4-n)!](0.4!)^n$

$= 1/1 - (4 \times 0.2) + (4 \times 3 \times 0.2^2) + (4 \times 3 \times 2 \times 0.2^3) + (4 \times 3 \times 2 \times 1 \times 0.2^4) = 0.4$ i.e 40 percent of the time the system is empty and 60% of the time the system is busy.

(a) Expected number of breakdown machines in the system $= E(n) = m = (\mu/\lambda)(1-P)$

$= 4 - (1/0.2)(1-0.4) = 4 - 5 \times 0.6 = 4 - 3 = 1$

(b) Expected break down cost per day of 8 hours

i.e ~~...~~ $8 \times$ (expected number of break down machines)(R, 25 per hour) =

$8 \times 1 \times 25 =$ Rs 200/day

(c) When there are two machines each servicing two machines $m = 2$, $P_0 =$

$P_0 = 1/\sum_{n=0}^{2} [2!/(2-n)!](0.2/1)^n = 1/1 + (2 \times 0.2) + (2 \times 1 \times 0.2^2) = 1/1.4 = 0.68$ i.e 68% of the system is idle. It is important that each mechanic with his two machines constitutes a separate system with no interplay. Expected number of machines in the system $= m - (\mu/\lambda) \times (1 - P_0) = 2 - (1/0.2 \times (1-0.68))$

$= 0.4$

CSC 314

4a) $\lambda = 10$ cars per hour

$\mu = 60/6 = 12$ cars per hour

$\rho = \lambda/\mu = 5/6 < 1$

$P_0 = 1 - \rho = 1/6$

Probability of having n customers in system :-

$P_n = (1-\rho)\rho^n$

$= (1/6 \times 5/6)$

A) Probability that system has less than 2 cars or 2 cars:

$P_0 + P_1 + P_2 = 1/6 + (1/6 \times 5/6)^1 + (1/6 \times 5/6)^2 = 0.3248$

B) Probability that system has 3 cars

$P_3 = (1/6 \times 5/6)^3 = 0.0026.$

C) $Wq = \dfrac{\lambda}{\mu(\mu-\lambda)} = \dfrac{10}{12(12-10)} = \dfrac{5}{12}$

5) Expand waiting time $= Wq = 1/\lambda\, Lq = \dfrac{\lambda}{\mu(\mu-\lambda)} = \dfrac{1/40}{1/25(1/25 - 1/40)}$

$= \dfrac{1/40}{1/25(3/200)}$

$= \dfrac{41.6667 \text{ minutes}}{2} = 20.8$ minutes.

6a) $P_0 = 1 - \rho$

$\rho = 1/3 \therefore P_0 = 2/3 = 66.6\%$ Probability of not having to wait.

c) Average length of a queue formed from time to time

$\therefore 1/1-\rho = \dfrac{1}{1-(1/3)} = \dfrac{1}{2/3} = 1.5$

(B) Expected percentage of idle time for a girl

$= P_0 = 1 - \rho = 1 - \frac{1}{3}$

$\therefore P_0 = \frac{2}{3}$

$\therefore$ Percentage of idle Time for each girl $=$

$\frac{2}{3} \times 100 = 67\%$.

(C) Expected length of customer waiting time:

$E(wq \mid ws > 0)$:

$E(wq \mid ws > 0) = \frac{1}{CM - \lambda} = \frac{1}{2 \times 15 - 10} = \frac{1}{20}$ hrs

$= \frac{1}{20} \times 60$ min

$= 3$ mins.

Therefore expected breakdown per day

$= 8 \times 0.4 \times$ number of ~~machines~~ mechanics or machines in system

$= 8 \times 0.4 \times 2 = 6.4$ hours per day.

Hence total cost

$=$ Rs $55 \times 2 + 6.4 \times$ Rs $25 =$ Rs $(110 + 160)$

$=$ Rs 270 per day.

But total cost with one mechanic Rs $(55 + 200)$

$=$ Rs 255/ per day, which is cheaper.

$\therefore$ hence using two mechanics is not economically advisable.

(20) $\lambda = 35 mins = 1/35 = 0.0286$

$\mu = 25 mins$

$\therefore P = \lambda/\mu = 0.0286/25 = 0.00114$

(A) idle $= 1 - P$

$= 1 - 0.00114$

$= 0.99886$ min

≈ 1 min

(B) $\lambda^2 / \mu(\mu - \lambda)$

$= (0.0286)^2 / 25(25 - 0.0286)$

$= 0.00081796 / 25(24.9714)$

$= 0.00081796 / 624.285$

$= 0.00$ min

$\therefore$ the average time a customer spent in the shop is 0 min.

(8) Incase Mr. Y is hired:

Average breakdown time of machine $= E_r = \frac{1}{\mu - \lambda} = \frac{6}{12-6} = \frac{1}{6}$ hrs

Total cost of breakdown = loss of productive time + cost of repair =

$= \frac{1}{6}$ hrs $\times$ Rs. 20 + $\frac{1}{2}$ hrs $\times$ Rs. 4 = Rs $\frac{9}{2}$

$\Rightarrow$ Rs. 4.5

(15) Given $\lambda = 10/hr$ Number of service $= 2$

Service Time $- \frac{1}{\mu} = 4$ mins

Service rate $= \mu = \frac{60}{4} = 15/hrs$

$$\therefore P_0 = \frac{1}{\sum_{n=0}^{c-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{c!}\left(\frac{\lambda}{\mu}\right)^c \left(\frac{c\mu}{c\mu - \lambda}\right)}$$

$$= \frac{1}{\sum_{n=0}^{2-1} \frac{1}{n!}\left(\frac{10}{15}\right)^n + \frac{1}{2!}\left(\frac{10}{15}\right)^2 \left(\frac{2\times 15}{2(15) - 10}\right)}$$

$$= \frac{1}{\sum_{n=0} \frac{1}{n!}\left(\frac{10}{15}\right)^n + \frac{1}{2}\left(\frac{10}{15}\right)^2 \cdot \frac{30}{20}}$$

$$= \frac{1}{\frac{1}{0!}\left(\frac{2}{3}\right)^0 + \frac{1}{1!}\left(\frac{2}{3}\right)^1 + \frac{1}{2}\left(\frac{2}{3}\right)^2\left(\frac{3}{2}\right)}$$

$$= \frac{1}{1 + \frac{2}{3} + \frac{1}{2} \times \frac{4}{9} \times \frac{3}{2}} = \frac{1}{1 + \frac{2}{3} + \frac{1}{3}}$$

$$= \frac{1}{\frac{3+2+1}{3}} \Rightarrow \frac{1}{6/3} \Rightarrow \frac{3}{6} \Rightarrow \frac{1}{2}$$

$\therefore P_0 = \frac{1}{2}$

(a) Probability of a customer having to wait for service:

$P(N \geq 2)$:

$$P(N \geq C) = \frac{\left(\frac{\lambda}{\mu}\right)^c}{c!\left(1 - \frac{\lambda}{c\mu}\right)} P_0 \Rightarrow \frac{\left(\frac{10}{15}\right)^2 \left(\frac{1}{2}\right)}{2!\left(1 - \frac{10}{2\times 15}\right)} = \frac{\left(\frac{2}{3}\right)^2 (\ldots)}{2(\ldots)}$$

$= \frac{4}{9} \times \frac{1}{4} \times \frac{3}{2} = \frac{1}{6}$

(18) $4/\lambda =$

$4/\mu$

$\rho = \lambda/\mu = 4/\lambda / 4\mu$

$= 4/\lambda \div 4/\mu$

$= \cancel{4}/\lambda \times \mu/\cancel{4}$

$= \mu/\lambda$

$\therefore \rho/1-\rho = \dfrac{\mu/\lambda}{1-\mu/\lambda}$

$= \dfrac{\mu/\lambda}{1-\mu/\lambda}$

$= 1-\mu/\lambda$

$= \dfrac{\lambda-\mu}{\lambda}$

$\therefore \dfrac{\mu/\lambda}{\lambda-\mu/\lambda} = \dfrac{\mu}{\lambda} \div \dfrac{(\lambda-\mu)}{\lambda}$

$\dfrac{\mu}{\cancel{\lambda}} \times \dfrac{\cancel{\lambda}}{(\lambda-\mu)}$

$= \dfrac{\mu}{\lambda-\mu}$

(a) No of customers in queue:

note: $\lambda = 9/5$; $\mu = 10/5$; $\rho = 9/10$

$$L_q = L_s - \rho = \frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{9/5}{10/5(10/5 - 9/5)}$$

$$= 9/2$$

(B) Probability of having more than 10 customers in the queue

$P(N>n) = \rho^{n+1}$

$n = 10$

$P(N>10) = \rho^{11} = p^{11} = (9/10)^{11}$

$= 0.3138$

(C) Customers to queue more than 2 mins

$P(w_q > t) = \lambda/\mu \, e^{-(\mu-\lambda)}$

$P(w_q > 2)\ 9/10\ e^{-(10/5 - 9/5)}$

$= 0.7368.$

(10): μ λ	Good weather 60 20	Bad weather 30 20
(a) average no of plane in queue; $\lambda^2/\mu \times 1/\mu-\lambda$	$\frac{400}{60} \times 1/40$ $= 1/6$	$\frac{400}{30} \times 1/10$ $= 4/3$
(b) average wait time in the system $1/\mu-\lambda$	$1/(60-20) = 1/40$ $= 1.5$ min	$1/(30-20) = 1/10$ hours $\Rightarrow$ 6 mins

(11) $\lambda = 25$ customers per hour; $\mu = \frac{3600}{120} = 30$ cust/hr

Average waiting Time of customers in queue $= \frac{\lambda}{\mu(\mu-\lambda)} = \frac{25}{30(30-25)}$

$= 1/6$

(12) $\lambda = 4$ cust/hr $\qquad \rho = \lambda/\mu = 4/10 = 0.4$

$\mu = 60/6 = 10$ cust/hr

(a) $P_0 = 1 - \rho = 0.6$

(b) Probability of at least one customer in queue $= 1 - (1-\rho) = 0.4$

(c) average no of customers: $L_s = \lambda/(\mu - \lambda) = 4/(10-4) = 2/3$

(d) average time spent including service: $W_s = 1/(\mu - \lambda) = 1/(10-4) = 1/6$ hrs $= 0$-10 mins

(13) $\lambda = 2$ belts per shift $\qquad \rho = 2/5$

$\mu = 5$ belts per shift

(a) a fraction of time system is empty - $P_0 = 1 - \rho = 1 - 2/5 = 2/3$

(b) average no of belts in the system: $L_s = \lambda/(\mu - \lambda) = 2/(5-2) = 2/3$

(c) waiting time in the queue for an arrival: $W_q = \lambda/\mu(\mu - \lambda)$

$= 2/5(3) = 2/15$

(d) average waiting time plus servicing time

$W_s = 1/(\mu - \lambda) = 1/(5-2) = 1/3$ shift

(14) $\lambda = 6$ $\qquad M_r Y = \mu_2 = 12/hr$

$M_r \lambda = \mu_1 = 8$

(a) In case Mr λ is hired

average breakdown time of machine

$E_r = \dfrac{1}{\mu - \lambda} = \dfrac{1}{8-6} = \dfrac{1}{2}$ hrs

Total cost of breakdown = loss of productive time + cost + cost of repair

$= 1/2$ hrs $\times$ Rs. $20 + 1/8$ hrs $\times$ Rs. $10 =$ Rs. $45/4$

$\Rightarrow$ Rs. 11.25

(5) $P(wq > t) = \lambda/M \, e^{-(M-\lambda)}$

$t = 10$

$$P(wq > 10) = \frac{1/12}{1/4} e^{-(1/4 - 1/12)} = 0.282$$

(6) $WS = 1/\lambda \, LS = 1/(M-\lambda) = 1/(1/4 - 1/12) = 6$

(7) $\lambda = 15$ customers per hour ; $M = ?$

Probability that the customer will not have to wait more than 12 mins = 0.9.

Since λ is in per hour, $t = 12/60 = 1/5$ hrs

$P(wq \leq 1/5) = 0.9$

recall $P = 1 - e$

$1 - e(wq > 1/5) = 0.9$

$P(wq > 1/5) = 1 - 0.9 = 0.1$

recall $P(wq > e) = \lambda/M \, e^{-(M-\lambda)}$

$\therefore \lambda/M \, e^{-(M-\lambda)t}$

$= 0.1$

$15/M \, e^{-(M-15)1/5} = 0.1$

$e^{(15-M)1/5} = \frac{0.1M}{1.5} = M/150$

$(15-M)1/5 = \log M/150$

$3 - 1/5 M = \log M - \log 150$

$1/5 M + \log M = 3 + \log 150$

$0.2m + \log M = 5.176$