19/MHS01/424 MBBS

UNUANE-HENRY OSEREMHEN MAT 104 – INTEGRATION

1 $\int \frac{2x}{\sqrt{4x^2-1}}\,dx$

Soln

$\int \frac{2x}{\sqrt{4x^2-1}}\,dx = \int \frac{2x}{u}\,dx$

Let $u = \sqrt{4x^2-1}$

$\therefore u^2 = 4x^2 - 1$

$2u\,du = 8x\,dx$

$\therefore \frac{2u\,du}{8x} = dx$

$\therefore dx = \frac{u\,du}{4x}$

$\therefore \int \frac{2x}{u} \times \frac{u\,du}{4x} = \int \frac{1}{2}\,du$

$\int \frac{1}{2}\,du = \left[\frac{1\,u^{0+1}}{2\times(0+1)} + C\right]$

$= \frac{1}{2}u + C$ But $u = \sqrt{4x^2-1}$

$\therefore = \frac{1}{2}\sqrt{4x^2-1} + C$

2 $\int \frac{\sin^{-1}x}{\sqrt{1-x^2}}\,dx$

Let $u = \sin^{-1}x$

$\therefore \int \frac{\sin^{-1}x}{\sqrt{1-x^2}}\,dx = \int \frac{u}{\sqrt{1-x^2}}\,dx$

$\frac{du}{dx} = \frac{1}{\sqrt{1-x^2}}$

$\therefore \frac{dx}{du} = \sqrt{1-x^2}$

$\therefore dx = \sqrt{1-x^2}\,du$

$= \int \frac{u}{\sqrt{1-x^2}} \times \sqrt{1-x^2}\,du$

$$= \int u \, du$$

$$= \frac{u^{1+1}}{1+1} + C$$

$$= \frac{u^2}{2} + C \quad \text{where } u = \sin^{-1}x$$

$$= \frac{(\sin^{-1}x)^2}{2} + C$$

3 $\int (\tan x)^6 \sec^2 x \, dx$

Let $u = \tan x$

$$\frac{du}{dx} = \sec^2 x$$

$$\therefore \frac{dx}{du} = \frac{1}{\sec^2 x}$$

$$\therefore dx = \frac{du}{\sec^2 x}$$

$$\int (\tan x)^6 \sec^2 x \, dx = \int u^6 \sec^2 x \, dx$$

$$\int u^6 \sec^2 dx = \int u^6 \cancel{\sec^2 x} \cdot \frac{du}{\cancel{\sec^2 x}}$$

$$= \int u^6 \, du$$

$$= \frac{u^{6+1}}{6+1} + C$$

$$= \frac{u^7}{7} + C \quad \text{where } u = \tan x$$

$$= \frac{(\tan x)^7}{7} + C$$