

$$1) \int \frac{2x}{\sqrt{4x^2-1}} dx$$

So

$$\text{Let } u = 4x^2 - 1$$

$$\left(\frac{u-1}{4}\right)^{1/2} = x$$

$$x = \left(\frac{u-1}{4}\right)^{1/2} \Rightarrow \frac{(u-1)^{1/2}}{2}$$

$$\frac{dx}{du} = \frac{1}{4(u-1)^{1/2}}$$

$$dx = du/4(u-1)^{1/2}$$

$$\int \frac{2x}{\sqrt{4x^2-1}} dx = \int \frac{2\left(\frac{u-1}{4}\right)^{1/2} \cdot 1}{2 \sqrt{u}}$$

$$= \int \frac{2(u-1)^{1/2} \cdot 1 \cdot du}{4 \cdot u^{1/2} \cdot 4(u-1)^{1/2}}$$

$$= \frac{1}{4} \int u^{-1/2} du$$

$$= \frac{1}{4} \left[\frac{u^{1/2}}{1/2} \right] + C$$

$$= \frac{2}{4} \left[(4x^2-1)^{1/2} \right] + C$$

$$= \int \frac{2x}{\sqrt{4x^2-1}} dx = \frac{1}{2} \left[\sqrt{4x^2-1} \right] + C$$

$$2) \int \frac{\sin^{-1} x}{\sqrt{1-x^2}} dx$$

$$\text{Let } u = \sin^{-1} x$$

$$\frac{du}{dx} = \frac{1}{\sqrt{1-x^2}}$$

$$dx = (\sqrt{1-x^2}) du$$

$$\int \frac{\sin^{-1} x}{\sqrt{1-x^2}} dx = \int \frac{u}{\sqrt{1-x^2}} \cdot \sqrt{1-x^2} du$$

$$= \int u du$$

$$= \frac{u^2}{2} + C$$

$$\int \frac{\sin^{-1} x}{\sqrt{1-x^2}} dx = \frac{(\sin^{-1} x)^2}{2} + C$$

$$3) \int (\tan x)^6 \sec^2 x dx$$

So

$$u = \tan x, \quad \frac{du}{dx} = \sec^2 x$$

$$dx = du/\sec^2 x$$

$$\int u^6 \cdot \sec^2 x \cdot \frac{du}{\sec^2 x}$$

$$= \frac{u^7}{7} + C$$

$$\int (\tan x)^6 \sec^2 x dx = \frac{(\tan x)^7}{7} + C$$