

Dept:
 Matrics
 No: 0

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MAT102

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Assignment

1. A Particle moves along a curve, $x = 7t^2$, $y = 6t^2 - 4t$, $z = t - 5$, where t is time. Find its Velocity

Solution

$$\text{Velocity} = \frac{dr}{dt}$$

Position Vector $r = xi + yj + zk$

$$\text{i.e } r = (7t^2)i + (6t^2 - 4t)j + (t - 5)k$$

$$\frac{dr}{dt} = (14t)i + (12t - 4)j + k$$

2. If $\bar{A} = i + 2j - 4k$, $\bar{B} = 2i - 3j + k$, $\bar{C} = 4j - 3k$. Find $A \times (B \times C)$

Solution

$$B \times C = \begin{vmatrix} i & j & k \\ 2 & -3 & 1 \\ 0 & 4 & -3 \end{vmatrix}$$

$$= i \begin{vmatrix} -3 & 1 \\ 4 & -3 \end{vmatrix} - j \begin{vmatrix} 2 & 1 \\ 0 & -3 \end{vmatrix} + k \begin{vmatrix} 2 & -3 \\ 0 & 4 \end{vmatrix}$$

$$= i(9 - 4) - j(-6 - 0) + k(8 + 0)$$

$$B \times C = 5i + 6j + 8k$$

Hence

$$A \times (B \times C) = \begin{vmatrix} i & j & k \\ 1 & 2 & -4 \\ 5 & 6 & 8 \end{vmatrix}$$

$$= i \begin{vmatrix} 2 & -4 \\ 6 & 8 \end{vmatrix} - j \begin{vmatrix} 1 & -4 \\ 5 & 8 \end{vmatrix} + k \begin{vmatrix} 1 & 2 \\ 5 & 6 \end{vmatrix}$$

$$= i(16 + 24) - j(8 + 20) + k(6 - 10)$$

$$= 40i - 28j - 4k$$

3. Given $B = 4\sin 3t \mathbf{i} + 4e^{3t} \mathbf{j} + 7t^3 \mathbf{k}$, find the integral of B with respect to t

Solution

$$B = 4\sin 3t \mathbf{i} + 4e^{3t} \mathbf{j} + 7t^3 \mathbf{k}$$

$$\int B dt = -\frac{1}{12} \cos 3t \mathbf{i} + \frac{1}{12} e^{3t} \mathbf{j} + \frac{7t^4}{4} \mathbf{k} + C$$

4. If $A = 7\mathbf{i} + 2\mathbf{j} - \mathbf{k}$, $B = 2\mathbf{i} + \mathbf{j} + 4\mathbf{k}$, $C = \mathbf{i} + \mathbf{j} + \mathbf{k}$, find $(A+C) \cdot (B-A)$

Solution

$$(A+C) = (7\mathbf{i} + 2\mathbf{j} - \mathbf{k}) + (\mathbf{i} + \mathbf{j} + \mathbf{k})$$

$$= 8\mathbf{i} + 3\mathbf{j}$$

$$(B-A) = (2\mathbf{i} + \mathbf{j} + 4\mathbf{k}) - (7\mathbf{i} + 2\mathbf{j} - \mathbf{k})$$

$$= -5\mathbf{i} - \mathbf{j} + 5\mathbf{k}$$

$$(A+C) \cdot (B-A) = (8\mathbf{i} + 3\mathbf{j}) \cdot (-5\mathbf{i} - \mathbf{j} + 5\mathbf{k})$$

$$= -40\mathbf{i} - 3\mathbf{j}$$

5. Find a unit vector tangent to the space curve $x = t$, $y = t^2$, $z = t^3$ at the point where $t = 1$

Solution

$$\mathbf{T} = \frac{d\mathbf{r}/dt}{|d\mathbf{r}/dt|}$$

$$\mathbf{r} = t\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$$

$$\frac{d\mathbf{r}}{dt} = \mathbf{i} + 2t\mathbf{j} + 3t^2\mathbf{k}$$

$$\text{at } t=1, \frac{d\mathbf{r}}{dt} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$$

$$\left| \frac{d\mathbf{r}}{dt} \right|_{t=1} = \sqrt{1^2 + 2^2 + 3^2} = 3.7$$

$$\text{Then } \mathbf{T} = \frac{\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}}{3.7}$$