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1 $\int \frac{2x}{\sqrt{4x^2-1}}\,dx$

$= 2\int \frac{dx}{\sqrt{4x^2-1}}$

Let $u = 4x^2 - 1$

$x = \left(\frac{u-1}{4}\right)^{1/2}$

$x = \frac{(u-1)^{1/2}}{2}$

$\frac{dx}{du} = \frac{1}{2}(u-1)^{-1/2} \times \frac{1}{2}$

$\frac{dx}{du} = \frac{1}{4}(u-1)^{-1/2}$

$\frac{dx}{du} = \frac{1}{4} \cdot \frac{1}{(u-1)^{1/2}}$

$\frac{dx}{du} = \frac{1}{4(u-1)^{1/2}}$

$dx = \frac{du}{4(u-1)^{1/2}}$

$= 2\int \frac{(u-1)^{1/2}}{2} \cdot u^{-1/2} \cdot \frac{du}{4(u-1)^{1/2}}$

$= 2\int \frac{u^{-1/2}\,du}{8}$

$= 2 \times \frac{1}{8}\int u^{-1/2}\,du$

$= \frac{1}{4}\int u^{-1/2}\,du$

$= \frac{1}{4} \times \frac{u^{\frac{1}{2}}}{\frac{1}{2}} + c$

$= \frac{1}{4} \times 2u^{1/2} + c$

$= \frac{1}{2}u^{1/2} + c$

$\therefore \int \frac{2x}{\sqrt{4x^2-1}}\,dx = \frac{\sqrt{4x^2-1}}{2} + c$

2 $\int \frac{\sin^{-1} x}{\sqrt{1-x^2}}\, dx$

$x = 1 \times \sin\theta$

$x = \sin\theta$ —— (i)

$x^2 = 1^2 \times \sin^2\theta$

$= \sin^2\theta$

$\frac{dx}{d\theta} = \cos\theta$

$dx = \cos\theta\, d\theta$

$1 - x^2 = 1 - \sin^2\theta$

Since $\cos^2\theta + \sin^2\theta = 1$

$\cos^2\theta = 1 - \sin^2\theta$

$1 - x^2 = \cos^2\theta$

$= \int \frac{\sin^{-1}(\sin\theta)\cos\theta\, d\theta}{\sqrt{\cos^2\theta}}$

$= \int \frac{\sin^{-1}(\sin\theta)\cos\theta\, d\theta}{\cos\theta}$

$= \int \sin^{-1}(\sin\theta)\, d\theta$

$= \frac{\theta^2}{2} + c$

From eqn (i), $\theta = \sin^{-1}(x)$

$\therefore \int \frac{\sin^{-1} x}{\sqrt{1-x^2}}\, dx = \frac{(\sin^{-1} x)^2}{2} + c$

3 $\int (\tan x)^6 \sec^2 x \, dx$

Let $u = \tan x$

$$\frac{du}{dx} = \sec^2 x$$

$$du = \sec^2 x \, dx$$

$$= \int u^6 \, du$$

$$= \frac{u^7}{7} + c$$

$$\therefore \int (\tan x)^6 \sec^2 x \, dx = \frac{(\tan x)^7}{7} + c$$