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19/ENG05/052
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Mechatronic Engineering

① If $A = 5i - 7j - 6k$, $B = j + 4k$, $C = 9i - 4j + k$, find $-8(A+B)\cdot(C-A)$

Soln

$-8(A+B) = -8[(5+0)i + (-7+1)j + (-6+4)k]$

$-8[5i - 6j - 2k]$

$-40i + 48j + 16k$

$(C-A) = [(9-5)i + (-4+7)j + (1+6)k]$

$4i + 3j + 7k$

$-8(A+B)\cdot(C-A) = -160 + 144 + 122$

$= 106$

② Find a unit vector to the space curve $x = -3t, y = t^2, z = 4$ at the point $t = 1$

$R = xi + yj + zk$

$R = (-3t)i + (t^2)j + (4t^3)k$

$\frac{dr}{dt} = -3i + (2t)j + (12t^2)k$

$\frac{dr}{dt}\Big|_{t=1} = -3i + 2j + 12k$

$\left|\frac{dr}{dt}\right| = \sqrt{9 + 4 + 144}$

$= \sqrt{157}$

$T = \frac{\frac{dr}{dt}}{\left|\frac{dr}{dt}\right|}$

$= \frac{-3i + 2j + 12k}{\sqrt{157}}$

3) A particle moves along a curve, $x = -8t^2$, $y = t^2 - 4t$, $z = t+1$ where t is time. Find its acceleration

$R = xi + yj + zk$

$R = (-8t^2)i + (t^2-4t)j + (t+1)k$

$\frac{dr}{dt} = (-16t)i + (2t+4)j + 1k$

$\frac{d^2r}{dt^2} = -16i + 2j$

4) If $A = i + 2j - 4k$, $B = 2i - 3j + k$, $C = 4j - 3k$, Find $(A \times B) \times C$

$$A \times B = \begin{vmatrix} i & j & k \\ 1 & 2 & -4 \\ 2 & -3 & 1 \end{vmatrix} = i\begin{vmatrix} 2 & -4 \\ -3 & 1 \end{vmatrix} - j\begin{vmatrix} 1 & -4 \\ 2 & 1 \end{vmatrix} + k\begin{vmatrix} 1 & 2 \\ 2 & -3 \end{vmatrix}$$

$$= i(2-12) - j(1+8) + k(-3-4)$$

$$= -10i - 9j - 7k$$

$$(A \times B) \times C = \begin{vmatrix} i & j & k \\ -10 & -9 & -7 \\ 0 & 4 & -3 \end{vmatrix} = i\begin{vmatrix} -9 & -7 \\ 4 & -3 \end{vmatrix} - j\begin{vmatrix} -10 & -7 \\ 0 & -3 \end{vmatrix} + k\begin{vmatrix} -10 & -9 \\ 0 & 4 \end{vmatrix}$$

$$= i(27+28) - j(30) + k(-40)$$

$$(A \times B) \times C = 55i - 30j - 40k$$

5) Given $R = 4\sin 3ti + 4e^{3t}j + 7t^3k$, find the integral of R wit respect to t from 0 to 1

$$\int_0^1 (4\sin 3ti + 4e^{3t}j + 7t^3k)\,dt$$

$$4\int_0^1 (\sin 3t)i\,dt + 4\int (e^{3t})j\,dt + \int (7t^3)k\,dt$$

$y = 3t$

$\frac{dy}{dt}$

(1) $I = \int_0^1 \sin 3t \, i \, dt$

$4 = 3t$

$\frac{dy}{dt} = 3$

$dt = \frac{dy}{3}$

$\therefore 4\int_0^1 (\sin 3t)\, i\, \frac{dy}{3} + 4\int_0^1 e^{3t} j\, dt + \int_0^1 (7t^3) k\, dt$

(2) $\frac{4}{3}\int_0^1 -\cos y \Big| + \frac{4}{3}\Big|_0^1 e^{3t} \Big| + \frac{7}{4}\Big|_0^1 t^4 \Big|$

$\frac{4}{3}\Big|_0^1 -\cos 3t \Big| + \frac{4}{3}\Big|_0^1 e^{3t} \Big| + \frac{7}{4}\Big|_0^1 t^4 \Big|$

$\frac{4}{3}\left[-\cos 3 - [-\cos 0]\right] + \frac{4}{3}\left[e^3 - e^0\right] + \frac{7}{4}\left[1^4 - 0^4\right]$

$\frac{4}{3}\left[-0.9986 + 1\right] + \frac{4}{3}\left[20.09 - 1\right] + \frac{7}{4}\left[1\right]$

$\frac{4}{3} \cdot 0.00186 + 25.453 + 1.75$

27.205