1. $\int \frac{2x}{\sqrt{4x^2-1}}\,dx$

$\int \frac{2x}{(4x^2-1)^{1/2}}\,dx$

Let $u = 4x^2 - 1$

$\frac{du}{dx} = 8x$

$dx = \frac{du}{8x}$

$\int \frac{2x}{u^{1/2}} \times \frac{du}{8x}$

$\int \frac{2x}{8x\,u^{1/2}} \times du$

$\frac{2}{8}\int u^{1/2} \times du$

$\frac{1}{4}\int u^{-1/2}\,du$

$= \frac{1}{4}\left[\frac{u^{-1/2+1}}{-1/2+1}\right] + c$

$= \frac{1}{4}\left[\frac{u^{+1/2}}{+1/2}\right] + c$

$= \frac{1}{4} \times 2\left(u^{1/2}\right) + c$

$= \frac{1}{2} \times \sqrt{u} + c$

$= \frac{\sqrt{4x^2-1}}{2} + c$

$\therefore \int \frac{2x}{\sqrt{4x^2-1}}\,dx = \frac{\sqrt{4x^2-1}}{2} + c$

2. $\int \frac{\sin^{-1}x}{\sqrt{1-x^2}}\,dx$

$$\int \frac{\sin^{-1}x}{(1-x^2)^{1/2}}\,dx$$

Let $u = \sin^{-1}x$

$$\frac{du}{dx} = (1-x^2)^{-1/2}$$

$$dx = (1-x^2)^{1/2}\,du$$

$$\int \frac{u}{(1-x^2)^{1/2}} \times (1-x^2)^{1/2}\,du$$

$$\int u\,du$$

$$\frac{u^{1+1}}{1+1} + C$$

$$\frac{u^2}{2} + C$$

3. $\int (\tan x)^6 \sec^2 x \, dx$

Let $u = \tan x$

$\frac{du}{dx} = \sec^2 x$

$dx = \frac{du}{\sec^2 x}$

$\int u^6 \times \sec^2 x \times \frac{du}{\sec^2 x}$

$\int u^6 \, du$

$= \frac{u^7}{7} + C$

$= \frac{(\tan x)^7}{7} + C$

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19/MHS01/078
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MAT 104