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Course: MAT104

Assignment

1. $\int \frac{2x}{\sqrt{4x^2-1}} dx$

Solution

$$\int \frac{2x}{\sqrt{4x^2-1}} dx$$

$$\text{Let } f = 4x^2 - 1$$

$$\frac{df}{dx} = 8x$$

$$dx = \frac{df}{8x}$$

$$\int \frac{2x}{\sqrt{4x^2-1}} dx = \int \frac{2x}{\sqrt{f}} \cdot \frac{df}{8x}$$

$$\int \frac{2x}{\sqrt{4x^2-1}} dx = \frac{1}{4} \int \frac{df}{\sqrt{f}}$$

$$\int \frac{2x}{\sqrt{4x^2-1}} dx = \frac{1}{4} \int \frac{1}{\sqrt{f}} df$$

$$\int \frac{2x}{\sqrt{4x^2-1}} dx = \frac{1}{4} \int \frac{1}{f^{1/2}} df$$

$$\int \frac{2x}{\sqrt{4x^2-1}} dx = \frac{1}{4} \int f^{-1/2} df$$

$$\int \frac{2x}{\sqrt{4x^2-1}} dx = \frac{1}{4} \left[\frac{f^{-1/2+1}}{-1/2+1} \right] + C$$

$$\int \frac{2x}{\sqrt{4x^2-1}} dx = \frac{1}{4} \left[\frac{f^{1/2}}{1/2} \right] + C$$

$$\int \frac{2x}{\sqrt{4x^2-1}} dx = \frac{1}{4} \left[\frac{f^{1/2} \cdot 2}{1} \right] + C$$

$$\int \frac{2x}{\sqrt{4x^2-1}} dx = \frac{1}{4} \left[f^{1/2} \times 2 \right] + C$$

$$\int \frac{2x}{\sqrt{4x^2-1}} dx = \frac{1}{2} \sqrt{f} + C$$

$$\int \frac{2x}{\sqrt{4x^2-1}} dx = \frac{1}{2} \sqrt{4x^2-1} + C$$

where C is the constant of integration

2.) $\int \frac{\sin^{-1} x}{\sqrt{1-x^2}}$

$$\int \frac{\sin^{-1} x}{\sqrt{1-x^2}}$$

Let $\theta = \sin^{-1} x$

$$\frac{d\theta}{dx}$$

$$dx = \frac{d\theta}{\cos \theta}$$

$$dx = \frac{d\theta}{\sqrt{1-\sin^2 \theta}}$$

$$\int \frac{\sin^{-1} x}{\sqrt{1-x^2}}$$

$$= \frac{1}{2} \int \frac{\theta}{\cos \theta} \cdot \frac{d\theta}{\sqrt{1-\sin^2 \theta}}$$

$$= \frac{1}{2} \int \frac{\theta}{\cos^2 \theta} d\theta$$

$$= \frac{1}{2} \int \theta \sec^2 \theta d\theta$$

$$x = \sin \theta$$

$$\frac{dx}{d\theta} = \cos \theta$$

$$d\theta = \frac{dx}{\cos \theta}$$

$$\frac{dx}{\sqrt{1-x^2}} = \frac{d\theta}{\sqrt{1-\sin^2 \theta}}$$

$$= \frac{d\theta}{\cos \theta}$$

$$\sqrt{1-x^2} = \cos \theta$$

$$\int \frac{\sin^{-1} x}{\sqrt{1-x^2}}$$

$$= \int \frac{\theta}{\cos \theta} \cdot \frac{d\theta}{\cos \theta}$$

$$= \int \theta \sec^2 \theta d\theta$$

$$x = \sin \theta$$

$$x^2 = \sin^2 \theta$$

$$dx = \cos \theta d\theta$$

$$d\theta = \frac{dx}{\cos \theta}$$

$$dx = \cos \theta d\theta$$

$$1-x^2 = \sqrt{1-\sin^2 \theta}$$

$$1-x^2 = \cos \theta$$

$$\text{Recall the}$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$2. \int \frac{\sin^{-1} x}{\sqrt{1-x^2}} dx$$

Solution

$$\int \frac{\sin^{-1} x}{\sqrt{1-x^2}} dx$$

$$\text{Let } a = \sin^{-1} x$$

$$\frac{da}{dx} = \frac{1}{\sqrt{1-x^2}}$$

$$dx = \sqrt{1-x^2} da$$

$$\int \frac{\sin^{-1} x}{\sqrt{1-x^2}} dx = \int \frac{a}{\sqrt{1-x^2}} \cdot \sqrt{1-x^2} da$$

$$\int \frac{\sin^{-1} x}{\sqrt{1-x^2}} = \int a da$$

$$\int \frac{\sin^{-1} x}{\sqrt{1-x^2}} = \frac{a^{1+1}}{1+1} + C$$

$$\int \frac{\sin^{-1} x}{\sqrt{1-x^2}} = \frac{a^2}{2} + C$$

$$\int \frac{\sin^{-1} x}{\sqrt{1-x^2}} = \frac{(\sin^{-1} x)^2}{2} + C$$

$$3. \int (\tan x)^6 \sec^2 x dx$$

Solution

$$\int (\tan x)^6 \sec^2 x dx$$

$$\text{Let } v = \tan x$$

$$\frac{dv}{dx} = \sec^2 x$$

$$dx = \frac{dv}{\sec^2 x}$$

$$dx = \frac{dv}{\sec^2 x}$$

$$\sec^2 x$$

$$\int (\tan x)^6 \sec^2 x dx = \int \frac{v^6 \sec^2 x \cdot dv}{\sec^2 x}$$

$$\int (\tan x)^6 \sec^2 x dx = \int v^6 dv$$

$$\int (\tan x)^6 \sec^2 x dx = \frac{v^{6+1}}{6+1} + C$$

$$\int (\tan x)^6 \sec^2 x \, dx = \frac{x^7}{7} + C$$

$$\int (\tan x)^6 \sec^2 x \, dx = \frac{(\tan x)^7}{7} + C$$