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Question 1

Derivative of the following from first principle;

a. $\frac{dy}{dx}$ of $\sin(3/x^2)$

$$y = \sin\left(\frac{3}{x^2}\right)$$

$$(\Delta y + y) = \sin\frac{3}{(\Delta x + x)^2}$$

$$\Delta y = \sin\left(\frac{3}{(\Delta x + x)^2}\right) - \sin\frac{3}{x^2}$$

$$\Delta y = \sin\left(\frac{3}{((\Delta x)^2 + 2x\Delta x + x^2)}\right) - \sin\frac{3}{x^2}$$

using trig identity

$$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$$

$$\frac{A+B}{2} = \frac{\frac{3}{(\Delta x)^2 + 2x\Delta x + x^2)} + \frac{3}{x^2}}{2} = \frac{3(x^2) + 3((\Delta x)^2 + 2x\Delta x + x^2)}{2x^2((\Delta x)^2 + 2x\Delta x + x^2)}$$

$$\frac{A+B}{2} = \frac{3x^2 + 3(\Delta x)^2 + 6x\Delta x + 3x^2}{2x^2((\Delta x)^2 + 2x\Delta x + x^2)} = \frac{\frac{3}{2}x^2 + \frac{3}{2}(\Delta x)^2 + 3x\Delta x + \frac{3}{2}x^2}{x^2((\Delta x)^2 + 2x\Delta x + x^2)}$$

$$= \frac{3x^2 + \frac{3}{2}(\Delta x)^2 + 3x\Delta x}{x^2((\Delta x)^2 + 2x\Delta x + x^2)}$$

$$\frac{A-B}{2} = \frac{\dfrac{3}{((\Delta x)^2+2x\Delta x+x^2)} - \dfrac{3}{x^2}}{2} = \frac{\dfrac{3x^2-3((\Delta x)^2+2x\Delta x+x^2)}{x^2((\Delta x)^2+2x\Delta x+x^2)}}{2}$$

$$= \frac{3x^2-3(\Delta x)^2-6x\Delta x-3x^2}{2(x^2(\Delta x)^2+2x\Delta x+x^2))}$$

$$= \frac{-\frac{3}{2}(\Delta x)^2-3x\Delta x}{(x^2(\Delta x)^2+2x\Delta x+x^2))}$$

$$\Delta y = 2\cos\left(\frac{3x^2+\frac{3}{2}(\Delta x)^2+3x\Delta x}{x^2((\Delta x)^2+2x\Delta x+x^2)}\right)\sin\left(\frac{-\frac{3}{2}(\Delta x)^2-3x\Delta x}{x^2(\Delta x^2+2x\Delta x+x^2)}\right)$$

$$\frac{\Delta y}{\Delta x} = \frac{2\cos\left(\dfrac{3x^2+\frac{3}{2}(\Delta x)^2+3x\Delta x}{x^2((\Delta x)^2+2x\Delta x+x^2)}\right)\sin\left(\dfrac{-\frac{3}{2}(\Delta x)^2-3x\Delta x}{x^2((\Delta x)^2+2x\Delta x+x^2}\right)}{\Delta x}$$

$$\frac{\Delta y}{\Delta x} = \cos\left(\frac{3x^2+\frac{3}{2}(\Delta x)^2+3x\Delta x}{x^2((\Delta x)^2+2x\Delta x+x^2}\right) \frac{2\sin\left(\dfrac{-\frac{3}{2}(\Delta x)^2-3x\Delta x}{x^2((\Delta x)^2+2x\Delta x+x^2}\right)}{\Delta x}$$

multiply the denominator & numerator by

$$\frac{-\frac{3}{2}\Delta x - 3x}{x^2((\Delta x)^2+2x\Delta x+x^2)}$$

$$\frac{\Delta y}{\Delta x} = \cos\left(\frac{3x^2+\frac{3}{2}(\Delta x)^2+3x\Delta x}{x^2((\Delta x)^2+2x\Delta x+x^2}\right) 2\left(\frac{-\frac{3}{2}\Delta x-3x}{x^2(\Delta x^2+2x\Delta x+x^2)}\right) \frac{\sin\left(\dfrac{-\frac{3}{2}(\Delta x)^2-3x\Delta x}{x^2((\Delta x)^2+2x\Delta x+x^2)}\right)}{\dfrac{-\frac{3}{2}\Delta x^2-3x\Delta x}{(x^2((\Delta x)^2+2x\Delta x+x^2)}}$$

Using Standard limit

$$\boxed{\frac{\sin\theta}{\theta} = 1}, \quad \theta = \frac{-\frac{3}{2}(\Delta x)^2 - 3x\Delta x}{x^2((\Delta x)^2+2x\Delta x+x^2)} \quad \therefore \frac{\sin\left(\frac{-\frac{3}{2}(\Delta x)^2 - 3x\Delta x}{x^2(\Delta x+2x\Delta x+x^2)}\right)}{\frac{-\frac{3}{2}(\Delta x)^2 - 3x\Delta x}{x^2((\Delta x)^2+2x\Delta x+x^2)}} = 1$$

$$\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x} = \lim_{\Delta x\to 0} \cos\left(\frac{+3x^2+\frac{3}{2}(\Delta x)^2+3x\Delta x}{x^2((\Delta x)^2+2x\Delta x+x^2}\right) \lim_{\Delta x\to 0} 2\left(\frac{-\frac{3}{2}\Delta x - 3x}{x^2((\Delta x)^2+2x\Delta x+x^2)}\right) \times 1$$

$$\frac{dy}{dx} = \cos\left(\frac{3x^2+0}{x^2(0+0+x^2)}\right)\cdot 2\left(\frac{-3x}{x^2(0+0+x^2)}\right)$$

$$\frac{dy}{dx} = \cos\left(\frac{3x^2}{x^4}\right)\cdot\frac{-6x}{x^4}$$

$$\frac{dy}{dx} = \cos\left(\frac{3}{x^2}\right)\cdot\frac{-6}{x^3}$$

$$\therefore \frac{dy}{dx} = \frac{-6}{x^3}\cos\left(\frac{3}{x^2}\right)$$

(ii) $y = \frac{4}{x^3}$

$$(\Delta y+y) = \frac{4}{(\Delta x+x)^3}$$

$$(\Delta y+y)(\Delta x+x)^3 = 4$$

Expansion

$$(\Delta y+y)((\Delta x)^2+2x\Delta x+x^2)(\Delta x+x) = 4$$

$$(\Delta y+y)((\Delta x)^3+x(\Delta x)^2+2x(\Delta x)^2+2x^2\Delta x+x^2\Delta x+x^3) = 4$$

$$(\Delta y+y)((\Delta x)^3+3x(\Delta x)^2+3x^2\Delta x+x^3) = 4$$

$$\Delta y(\Delta x)^3 + 3x(\Delta x)^2\Delta y + 3x^2\Delta x\Delta y + x^3\Delta y + (\Delta x)^3 y + 3x(\Delta x)^2 y + 3x^2\Delta x y + x^3 y = 4 \longrightarrow *$$

input $y = \frac{4}{x^2}$ any where it occurs in the equation *

$$\Delta y(\Delta x)^3 + 3x(\Delta x)^2\Delta y + 3x^2\Delta x\Delta y + x^3\Delta y + (\Delta x)^3 \cdot \frac{4}{x^3} + 3\cancel{x}(\Delta x)^2 \cdot \frac{4}{\cancel{x^3}_{x^2}} + 3\cancel{x^2}\Delta x \cdot \frac{4}{\cancel{x^3}_{x}} + \cancel{x^3} \cdot \frac{4}{\cancel{x^3}}$$

$$\Delta y(\Delta x)^3 + 3x(\Delta x)^2\Delta y + 3x^2\Delta x\Delta y + x^3\Delta y + \frac{4(\Delta x)^3}{x^3} + \frac{3 \cdot 4(\Delta x)^2}{x^2} + \frac{3 \cdot 4\Delta x}{x} + 4 = 4$$

$$\Delta y(\Delta x)^3 + 3x(\Delta x)^2\Delta y + 3x^2\Delta x\Delta y + x^3\Delta y + \frac{4(\Delta x)^3}{x^3} + \frac{12(\Delta x)^2}{x^2} + \frac{12\Delta x}{x} = 4 - 4$$

$$\Delta y(\Delta x)^3 + 3x(\Delta x^2)\Delta y + 3x^2\Delta x\Delta y + x^3\Delta y + \frac{4(\Delta x)^3}{x^3} + \frac{12(\Delta x)^2}{x^2} + \frac{12\Delta x}{x} = 0$$

$$\Delta y(\Delta x)^3 + 3x(\Delta x)^2\Delta y + 3x^2\Delta x\Delta y + x^3\Delta y = -\frac{4(\Delta x)^3}{x^3} - \frac{12(\Delta x)^2}{x^2} - \frac{12\Delta x}{x}$$

factorizing the two sides

$$\Delta y((\Delta x)^3 + 3x(\Delta x)^2 + 3x^2\Delta x + x^3) = \Delta x\left(\frac{4(\Delta x)^2}{x^3} - \frac{12\Delta x}{x^2} - \frac{12}{x}\right)$$

Divide both sides by Δx

$$\frac{\Delta y}{\Delta x}((\Delta x)^3 + 3x(\Delta x)^2 + 3x^2\Delta x + x^3) = \frac{4(\Delta x)^2}{x^3} - \frac{12\Delta x}{x^2} - \frac{12}{x}$$

Divide both sides by $((\Delta x)^3 + 3x(\Delta x)^2 + 3x^2\Delta x + x^3)$

$$\frac{\Delta y}{\Delta x} = \frac{\frac{4(\Delta x)^2}{x^3} - \frac{12\Delta x}{x^2} - \frac{12}{x}}{(\Delta x)^3 + 3x(\Delta x)^2 + 3x^2\Delta x + x^3}$$

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{\frac{4(0)^2}{x^3} - \frac{12(0)}{x^2} - \frac{12}{x}}{0^3 + 3x(0)^2 + 3x^2(0) + x^3}$$

$$\therefore \frac{dy}{dx} = \frac{\frac{-12}{x}}{x^3}$$

$$\frac{dy}{dx} = \frac{-12}{x^4}$$

Question 2

Integrate the following

1) $\int \frac{1}{x^2+36} dx = \int \frac{1}{x^2+6^2} dx$

Let $x = 6\tan\theta$

$\therefore \frac{dx}{d\theta} = 6\sec^2\theta$

$dx = 6\sec^2\theta d\theta$

$$= \int \frac{1}{(6\tan\theta)^2+36} \cdot 6\sec^2\theta d\theta$$

$$= \int \frac{1}{36\tan^2\theta+36} \cdot 6\sec^2\theta d\theta$$

$$= \int \frac{1}{36(\tan^2\theta+1)} \cdot 6\sec^2\theta d\theta$$

$$= \int \frac{1}{36\sec^2\theta} \cdot 6\sec^2\theta d\theta$$

$$= \int \frac{1}{6} d\theta = \frac{1}{6}\int d\theta = \frac{1}{6}\cdot\theta$$

But, $\theta = \tan^{-1}\frac{x}{6}$

$$\therefore \int \frac{1}{x^2+36}\,dx = \frac{1}{6}\tan^{-1}\frac{x}{6} + c$$

(ii) $\int \frac{1}{x^2+13}\,dx$

Let $x = \sqrt{13}\tan\theta$, $\frac{dx}{d\theta} = \sqrt{13}\sec^2\theta$, $dx = \sqrt{13}\sec^2\theta\,d\theta$

$$= \int \frac{1}{(\sqrt{13}\tan\theta)^2+13} \cdot \sqrt{13}\sec^2\theta\,d\theta$$

$$= \int \frac{1}{13\tan^2\theta+13} \cdot \sqrt{13}\sec^2\theta\,d\theta$$

$$= \int \frac{1}{13(\tan^2\theta+1)} \cdot \sqrt{13}\sec^2\theta\,d\theta$$

$$= \int \frac{1}{13\sec^2\theta} \cdot \sqrt{13}\sec^2\theta\,d\theta$$

$$= \int \frac{\sqrt{13}}{13}\,d\theta = \frac{\sqrt{13}}{13}\int d\theta = \frac{\sqrt{13}}{13}\cdot\theta = \frac{1}{\sqrt{13}}\cdot\theta$$

But $\theta = \tan^{-1}\frac{x}{\sqrt{13}}$

$$\therefore \int \frac{1}{x^2+13}\,dx = \frac{1}{\sqrt{13}}\tan^{-1}\frac{x}{\sqrt{13}} + C$$