

1) $\int \frac{2x}{\sqrt{4x^2-1}} dx$

sol
let $u = \sqrt{4x^2-1} = (4x^2-1)^{1/2}$
 $\frac{du}{dx} = \frac{1}{2} (4x^2-1)^{-1/2} \cdot 8x$

$\frac{du}{dx} = 4x (4x^2-1)^{-1/2}$

$dx = \frac{du}{4x (4x^2-1)^{-1/2}} = \frac{(4x^2-1)^{1/2} du}{4x}$

$2 \int \frac{x}{u} dx = 2 \int \frac{x}{(4x^2-1)^{1/2}} \times \frac{(4x^2-1)^{1/2} du}{4x}$

$\Rightarrow \frac{1}{2} \int du$

$= \frac{1}{2} u + C = \frac{1}{2} \sqrt{4x^2-1} + C$

2) $\int \frac{\sin^{-1} x}{\sqrt{1-x^2}} dx$

$= \int \sin^{-1} x \cdot (1-x^2)^{-1/2} dx$

let $u = \sin^{-1} x$

$du = \frac{1}{\sqrt{1-x^2}} dx$

$\int u du = \frac{u^2}{2} + C$

$= \frac{(\sin^{-1} x)^2}{2} + C$

3. $\int (\tan x)^6 \sec^2 x dx$

let $u = \tan x$

$du = \sec^2 x dx$

we have

$\int u^6 du = \frac{u^7}{7} + C$

$= \frac{(\tan x)^7}{7} + C$