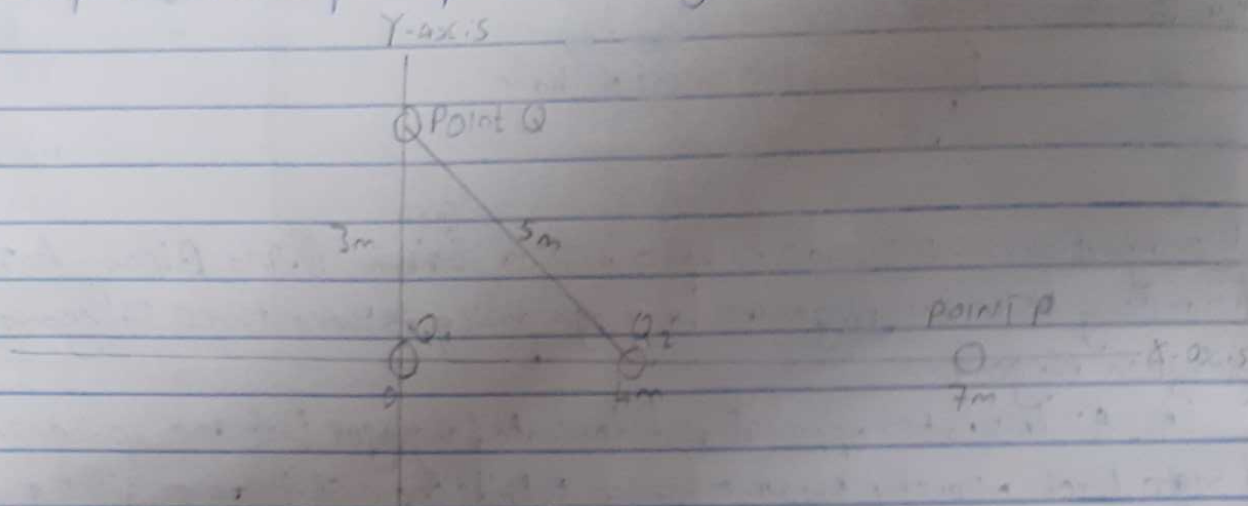


Section A

2 (a) An electric field is a region of space in which an electric charge will experience an electric force. While an electric field intensity can be defined as the force per unit charge.

(b)



(i)

$$q_1 = 8\mu\text{C}, q_2 = 12\mu\text{C}, r_1 = 7\text{m}, r_2 = 3\text{m}, k = 9 \times 10^9 \text{Nm}^2/\text{C}^2$$

$$E_1 = \frac{k \times q_1}{r_1^2} = \frac{9 \times 10^9 \times 8 \times 10^{-9}}{7^2} = 1.669 \text{N/C}$$

$$E_2 = \frac{k \times q_2}{r_2^2} = \frac{9 \times 10^9 \times 12 \times 10^{-9}}{3^2} = 12 \text{N/C}$$

| Vector | Angle | X-Component | Y-Component |
|--------|-------|-------------|-------------|
| 1.669 | 0 | 1.669 | 0 |
| 12 | 0 | 12 | 0 |
| | | 13.669 | 0 |

$$E = \sqrt{(\text{x-component})^2 + (\text{y-component})^2}$$

$$E = \sqrt{(13.669)^2 + 0} = 13.47 \text{N/C}$$

(ii)

$$q_1 = 8\mu\text{C}, q_2 = 12\mu\text{C}, r_1 = 3\text{m}, r_2 = 5\text{m}, k = 9 \times 10^9 \text{Nm}^2/\text{C}^2$$

$$E_1 = \frac{9 \times 10^9 \times 8 \times 10^{-9}}{3^2} = 8 \text{N/C}$$

$$E_2 = \frac{9 \times 10^9 \times 12 \times 10^{-9}}{5^2} = 4.32 \text{N/C}$$

| Vector | Angle | x-components | y-components |
|--------|--------|--------------|--------------|
| 8 | 90° | 0 | 8 |
| L 32 | 53.13° | 2.59 | 3.46 |
| | | 2.59 | 11.46 |

$$E = \sqrt{(x\text{-components})^2 + (y\text{-components})^2}$$

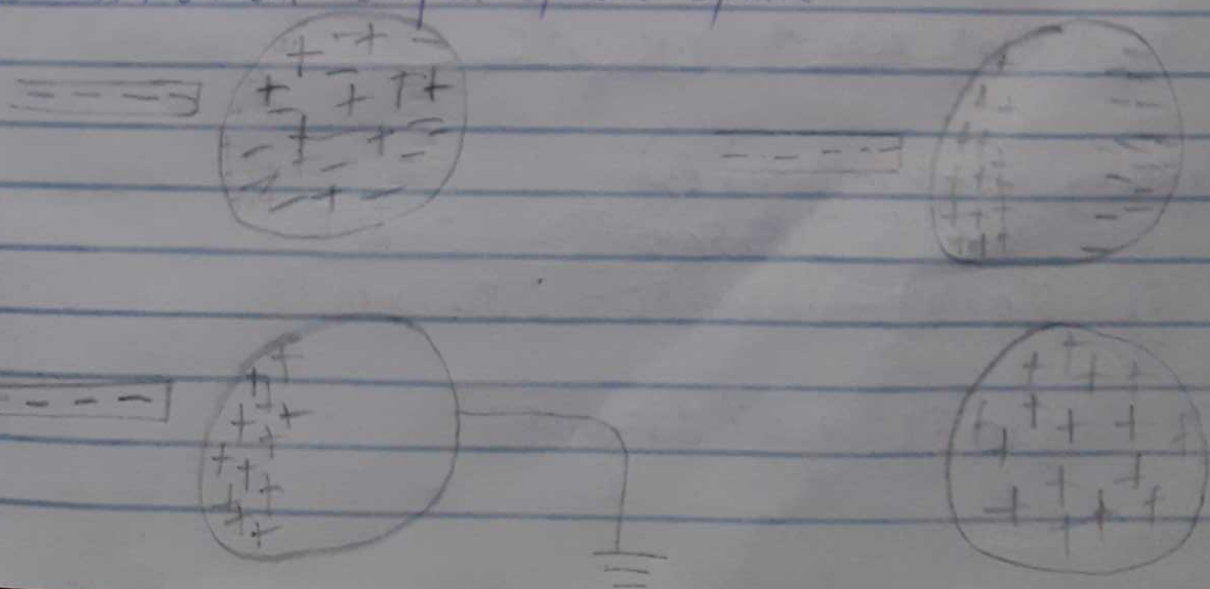
$$E = \sqrt{(2.59)^2 + (11.46)^2}$$

$$E = 11.75 \text{ N/C}$$

1 (a) Charging by Induction

Consider a negatively charged rubber rod brought near a neutral (uncharged) conducting sphere that is insulated so that there is no conducting path to ground as shown below. The repulsive force between the electrons in the rod and those in the ^{sphere} surface causes a redistribution of charges on the sphere, so that some electrons move to the side of the sphere furthest away from the rod. The region of the sphere nearest the negatively charged rod has an excess of positive charge because of the migration of electrons away from this location.

If a grounded conducting wire is then connected to the sphere, as in some of the electrons leave the sphere and travel to the earth. If the wire to the ground is then removed, the conducting sphere is left with an excess of induced positive charge. Finally, when the rubber rod is removed from the vicinity of the sphere, the induced positive charge remains on the ungrounded sphere and becomes uniformly distributed on the surface of the sphere.



$$\therefore 1 \text{ (b)} \quad q_1 + q_2 = 5.0 \times 10^{-5} \text{ C}, \quad q_1 = 5.0 \times 10^{-5} \text{ C} - q_2$$

$$F = 1.0 \text{ N}$$

$$r = 2.0 \text{ m}$$

$$k = 9 \times 10^9 \text{ Nm}^2/\text{C}^2$$

$$F = \frac{k q_1 q_2}{r^2} \Rightarrow 1.0 = \frac{9 \times 10^9 (q_1 q_2)}{(2)^2}$$

$$4 = 9 \times 10^9 (q_1 q_2)$$

$$(q_1 q_2) = \frac{9 \times 10^9}{4} \quad (q_1 q_2) = 4$$
$$\frac{9 \times 10^9}{9 \times 10^9}$$

$$q_1 q_2 = 4.44 \times 10^{-10} \quad \text{--- (2)}$$

$$q_1 + q_2 = 5 \quad \text{Recall } q_1 = 5.0 \times 10^{-5} \text{ C} - q_2 \quad \text{--- (1)}$$

Substituting eqn (1) into eqn (2)

$$(5.0 \times 10^{-5} - q_2) q_2 = 4.44 \times 10^{-10}$$

$$5.0 \times 10^{-5} q_2 - q_2^2 = 4.44 \times 10^{-10}$$

$$q_2^2 - 5.0 \times 10^{-5} q_2 + 4.44 \times 10^{-10} = 0$$

$$\text{Using } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\text{where; } a = 1, \quad b = -5.0 \times 10^{-5}, \quad c = 4.44 \times 10^{-10}$$

$$q = \frac{-(-5.0 \times 10^{-5}) \pm \sqrt{(-5.0 \times 10^{-5})^2 - 4(1)(4.44 \times 10^{-10})}}{2(1)}$$

$$q = \frac{5.0 \times 10^{-5} \pm 2.69 \times 10^{-5}}{2}$$

$$\text{For } q_1 = \frac{5.0 \times 10^{-5} + 2.69 \times 10^{-5}}{2} = 3.845 \times 10^{-5} \text{ C}$$

$$\text{For } q_2 = \frac{5.0 \times 10^{-5} - 2.69 \times 10^{-5}}{2} = 1.15 \times 10^{-5} \text{ C}$$

Replacing values into eqn (1)

$$q_1 = 5.0 \times 10^{-5} - 3.845 \times 10^{-5} \text{ C} = 1.15 \times 10^{-5} \text{ C}$$

$$q_2 = 5.0 \times 10^{-5} - 1.15 \times 10^{-5} \text{ C} = 3.845 \times 10^{-5} \text{ C}$$

$$\therefore q_1 = 1.15 \times 10^{-5} \text{ C}, \quad q_2 = 3.845 \times 10^{-5} \text{ C}$$

$$10) E_1 = \frac{kq_1}{r^2} = \frac{9 \times 10^9 \times 8 \times 10^{-6}}{1.1^2} = 5.9 \times 10^4$$

$$E_2 = \frac{kq_2}{r^2} = \frac{9 \times 10^9 \times 8 \times 10^{-6}}{1.1^2} = 5.9 \times 10^4$$

$$E_q = \frac{kq}{r^2} = \frac{9 \times 10^9 \times q}{1^2} = 9 \times 10^9 q$$

| Vector | Angle | X-Comp | Y-Comp |
|-------------------------|--------------|--------------------|-------------------|
| $E_1 = 5.9 \times 10^4$ | 63.4° | -2.6×10^4 | 5.3×10^4 |
| $E_2 = 5.9 \times 10^4$ | 63.4° | 2.6×10^4 | 5.3×10^4 |
| $E_q = 9 \times 10^9 q$ | 90° | 0 | $9 \times 10^9 q$ |

$$|E| = \sqrt{(E_x)^2 + (E_y)^2} = \sqrt{(0)^2 + (1.0 \times 10^5)^2 + (9 \times 10^9 q)^2}$$

$$|E| = \sqrt{(1.0 \times 10^5)^2 + (9 \times 10^9 q)^2}$$

$$|E| = 1.0 \times 10^5 + 9 \times 10^9 q$$

where $|E| = 0$

$$0 = 1.0 \times 10^5 + 9 \times 10^9 q$$

$$q = \frac{-1.0 \times 10^5}{9 \times 10^9} = -1.1 \times 10^{-5} \text{ C}$$

SECTION B

4(a) ~~Ang~~ Magnetic flux is defined as the strength of the magnetic field represented by lines of force. It is represented by the symbol Φ .

5) $m = 9.1 \times 10^{-31} \text{ kg}$, $r = 1.4 \times 10^{-7} \text{ m}$, $B = 3.5 \times 10^4 \text{ webster/m}^2$, $q = 1.6 \times 10^{-19} \text{ C}$

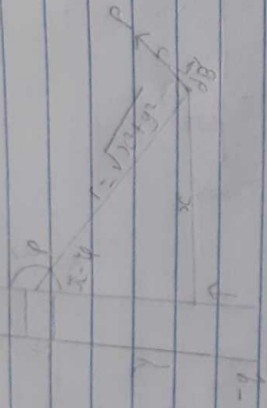
Cyclotron frequency, $f = \frac{qB}{m}$

$$f = \frac{1.6 \times 10^{-19} \times 3.5 \times 10^4}{9.11 \times 10^{-31}}$$

$$f = 6.147 \times 10^{14} \text{ rad/s}$$

5 a) Biot-Savart law states that the magnetic intensity at any point due to a steady current in an infinitely long straight wire is directly proportional to the current and inversely proportional to the distance from point to wire.

b) Magnetic field of a straight current carrying conductor.



Applying the Bio-Savart law, we can find the magnitude of the field

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{dl \sin \theta}{r^2}$$

$$\sin(\pi - \theta) = \sin \theta$$

$$\therefore B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{dl \sin(\pi - \theta)}{r^2}$$

From diagram, $r^2 = x^2 + y^2$ (Pythagoras theorem)
 $B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{dl \sin(\pi - \theta)}{x^2 + y^2}$ (*)

$$\text{But } \sin(\pi - \theta) = x \quad \text{--- (*)}$$

$$\frac{x}{\sqrt{x^2 + y^2}}$$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{dl \cdot x}{(x^2 + y^2)^{3/2}}$$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{dl \cdot x}{(x^2 + y^2)^{3/2}}$$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{x \, dy}{(x^2 + y^2)^{3/2}}$$

$$B = \frac{\mu_0 I x}{4\pi} \int_{-a}^a \frac{1}{(x^2 + y^2)^{3/2}} \, dy \quad \text{--- (*)}$$

$$\int \frac{dy}{(x^2 + y^2)^{3/2}} = \frac{1}{x^2} \frac{y}{(x^2 + y^2)^{1/2}}$$

$$B = \frac{\mu_0 I x}{4\pi} \left[\frac{y}{x^2 \sqrt{x^2 + y^2}} \right]_{-a}^a$$

$$B = \frac{\mu_0 I x}{4\pi} \left(\frac{2a}{x^2 \sqrt{x^2 + a^2}} \right)$$

$$B = \frac{\mu_0 I}{4\pi x} \left(\frac{2a}{\sqrt{x^2 + a^2}} \right)$$

When the length $2a$ of the conductor is very great in comparison to its distance x from point P , we consider it infinitely long. That is when a is much larger than x ,

$$\sqrt{x^2 + a^2} \approx a, \quad \text{as } a \rightarrow \infty$$

$$\therefore B = \frac{\mu_0 I}{2\pi x}$$

In a physical situation, we have axial symmetry about the y -axis. Thus, at all points in a circle of radius r around the conductor, the magnitude of B is

$$B = \frac{\mu_0 I}{2\pi r} \quad \text{--- (#)}$$

Equation (#) defines the magnitude of the magnetic field of flux density B near a long straight current-carrying conductor.