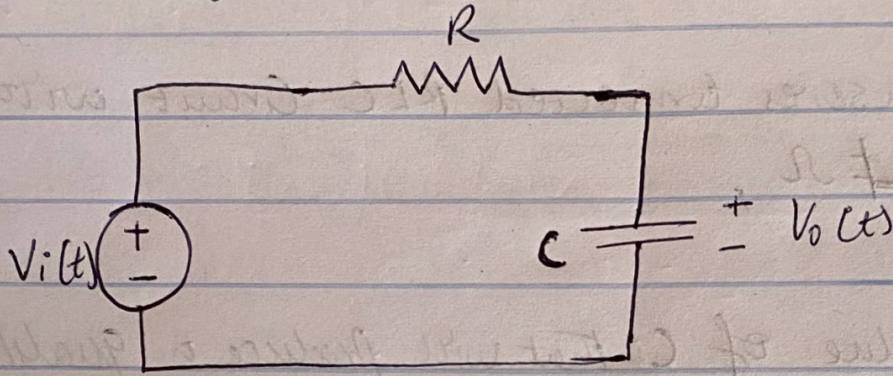


⇒ Determine the type of filter shown below, and show that its cut-off frequency is: $\omega_c = \frac{1}{RC}$



Soln

recall: $H(\omega) = \frac{V_o}{V_i}$

but $V_o = \frac{1}{sC}$, $V_i = R + \frac{1}{sC}$ but $s = j\omega$

Hence $H(\omega) = \frac{\frac{1}{sC}}{R + \frac{1}{sC}} \Rightarrow \frac{V_o}{V_i} = \left[\frac{\frac{1}{sC}}{R + \frac{1}{sC}} \right] \times V_i$

~~$\frac{RSC + 1}{sC} \times \frac{sC}{RSC + 1}$~~

$\frac{V_o}{V_i} = \frac{1}{sC} \times \frac{sC}{RSC + 1} = \frac{1}{RSC + 1}$

$$\omega^0 \quad H(\omega) = \frac{1}{RSC + 1} \quad \text{recall: } s = j\omega$$

$$H(0) = 1$$

$$H(\infty) = 0$$

\Rightarrow Hence: The cutoff frequency, ω_c is obtained by setting the magnitude of $H(\omega)$ to $\frac{1}{\sqrt{2}}$

$$|H(\omega)| = \left| \frac{1}{RSC + 1} \right| = \frac{1}{\sqrt{2}}$$

$$|H(\omega)| = \frac{\sqrt{1^2}}{\sqrt{1^2 + (j\omega RC)^2}} = \frac{1}{\sqrt{2}}$$

$$|H(\omega)| = \frac{1}{\sqrt{1 + \omega^2 R^2 C^2}} \quad \xrightarrow{\quad} \frac{1}{\sqrt{2}}$$

$$\sqrt{2} = \sqrt{1 + \omega^2 R^2 C^2} \quad (\text{squaring both sides})$$

$$2 = 1 + \omega^2 R^2 C^2$$

$$2 - 1 = \omega^2 R^2 C^2 \quad \Rightarrow \quad 1 = \omega^2 R^2 C^2$$

$$\omega^2 = \frac{1}{R^2 C^2}$$

$$\therefore \omega_c = \frac{1}{RC}$$

Therefore it is a low pass filter