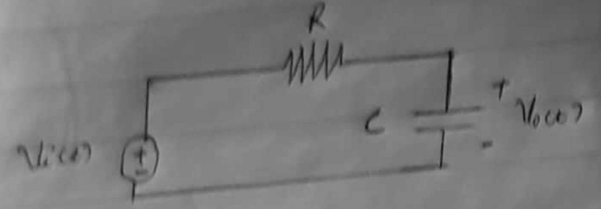
Ahon Tochukwu C.
17/ENG04/013
Elect / Elect Engineering
Circuit Theory

Question

Determine the type of filter shown below and show that its cut off frequency is $\omega_c = 1/RC$



$T_F = \frac{1}{j\omega C}$ $\quad V_o = \frac{1}{j\omega C}$

$V_i = R + 1/j\omega C$ $\quad H(\omega) = \frac{V_o}{V_i} = \frac{1/j\omega C}{R + j\omega C}$

$\frac{V_o}{V_i} = \left[\frac{1/j\omega C}{R + j\omega C}\right] V_i = \frac{1}{j\omega C} \times R + 1/j\omega C$

$= \frac{1}{j\omega RC + 1}$

when $(j\omega) = 0$ $\quad H = 1$ $\quad$ when $(\omega) \to \infty$ $\quad H = 0$

So therefore the cut off frequency ω_c is obtain the magnitude of $H(\omega)$ to $1/\sqrt{2}$ which is

$H(\omega) = \frac{1}{j\omega RC + 1} = \frac{1}{\sqrt{2}}$

$H(\omega) = \frac{\sqrt{1^2}}{\sqrt{1^2 + (j\omega RC)^2}} = \frac{1}{\sqrt{1 + (\omega^2 C^2 R^2)}} = \frac{1}{\sqrt{2}}$

$\sqrt{2} = \sqrt{1 + \omega^2 R^2 C^2}$

Squaring both sides

$2 = 1 + \omega^2 R^2 C^2$

$2 - 1 = \omega^2 R^2 C^2$

$1 = \omega^2 R^2 C^2$

$\omega^2 = \frac{1}{R^2 C^2}$

$\omega = \omega_c$

$\omega = \frac{1}{RC}$

It is a Low pass filter since its an RC circuit