DARE BENEDICT OLUBUKOLA

MECHANICAL ENGINEERING

19/ENG06/016 SERIAL NO.: 111

MAT 104 ASSIGNMENT

1 Determine the stationary point, Coordinate of the stationary point and nature of the Stationary point of the curve,

$$y = \frac{t^3 - t^2}{2 - 2t + 4}$$

Solution

$$y = \frac{t^3 - t^2}{2 - 2t + 4} \quad \text{— U} \qquad \text{— V}$$

$$\frac{dy}{dt} = \frac{V\frac{dv}{dt} - U\frac{dv}{dt}}{V^2}$$

$$= \frac{(2 - 2t + 4)(3t^2 - 2t) - (t^3 - t^2)(-2)}{(2 - 2t + 4)^2}$$

$$= \frac{6t^2 - 2t - 6t^3 + 4t^2 + 12t^2 - 8t + 2t^3 + 2t^2}{4 - 4t + 8 - 4t + 4t^2 - 8t + 8 - 8t + 16}$$

$$= \frac{-4t^3 + 24t^2 - 12t}{4t^2 - 24t + 36} = \frac{-4t(t^2 - 6t + 3)}{4t(t - 6 + 9)}$$

$$\therefore \frac{dy}{dt} = \frac{-(t^2 - 6t + 3)}{(t - 6 + 9)}$$

At Stationary point, $\frac{dy}{dt} = 0$

$$\therefore \frac{-(t^2 - 6t + 3)}{(t - 6 + 9)} = 0$$

$$\Rightarrow -(t^2 - 6t + 3) = 0$$

$$-t^2 + 6t - 3 = 0$$

Divide through by -1

$$t^2 - 6t + 3 = 0$$

$$t^2 - 6t = -3$$

$$t^2 - 6t + [-3]^2 = -3 + [-3]^2$$

$$[t - 3]^2 = -3 + 9$$

$$[t - 3]^2 = 6$$

$$t - 3 = \pm\sqrt{6}$$

$$t = 3 + 2.45 \text{ or } 3 - 2.45$$

$$t = 5.45 \text{ or } 0.55$$

★ Coordinate of Stationary Point,

At $t = 5.45$

$$y = \frac{(5.45)^3 - (5.45)^2}{2 - 2(5.45) + 4}$$

$$y = \frac{161.88 - 29.70}{2 - 10.9 + 4}$$

$$y = -26.98$$

At $t = 0.55$,

$$y = \frac{(0.55)^3 - (0.55)^2}{2 - 2(0.55) + 4}$$

$$y = \frac{0.17 - 0.30}{2 - 1.1 + 4}$$

$$y = -0.03$$

$\therefore$ Coordinate $\Rightarrow$ $(5.45, -26.98)$ and $(0.55, -0.03)$

Nature of Stationary point,

$$\frac{d^2y}{dt^2} = \frac{-(t^2 - 6t + 13)}{(t - 3 + 9)}$$

$$\frac{d^2y}{dt^2} = \frac{-t^2 + 6t + 15}{t^2 + 6t + 9}$$

At $t = 5.45$

$$\frac{d^2y}{dt^2} = \frac{-(5.45)^2 + 6(5.45) + 15}{(5.45)^2 + 6(5.45) + 9}$$

$$= \frac{-29.70 + 32.7 + 15}{29.70 + 32.7 + 9}$$

$$= 0.25$$

$\therefore$ At $(5.45, -26.98)$, we have a minimum point.

At $t = 0.55$

$$\frac{d^2y}{dt^2} = \frac{-(0.55)^2 + 6(0.55) + 15}{(0.55)^2 + 6(0.55) + 9}$$

$$= 1.43_{//}$$

$\therefore$ At $(0.55, -0.03)$ we have a minimum point.

2. If $2y^2 - 5x^4 - 2 - 7y^3 = 0$, find dy/dx.

Solution

~~$\frac{dy}{dx} = 4y\frac{dy}{dx} - 20x^3 - 21y^2\frac{dy}{dx}$~~

$$2y^2 - 5x^4 - 2 - 7y^3 = 0$$

$$4y\frac{dy}{dx} - 20x^3 - 21y^2\frac{dy}{dx} = 0$$

$$4y\frac{dy}{dx} - 21y^2\frac{dy}{dx} = 20x^3$$

$$\frac{dy}{dx}(4y - 21y^2) = 20x^3$$

$$\therefore \frac{dy}{dx} = \frac{20x^3}{4y - 21y^2}$$

3. Find $\frac{dy}{dx}$ if $4x^2 + 2xy^3 - 5y^2 = 0$ and evaluate ~~$\frac{dy}{dx}$~~ when $x = 1$ and $y = 2$

Solution

$$4x^2 + 2xy^3 - 5y^2 = 0$$

$$8x + 2y^3 + 3y^2\frac{dy}{dx}2x - 10y\frac{dy}{dx} = 0$$

$$6xy^2\frac{dy}{dx} - 10y\frac{dy}{dx} = -8x - 2y^3$$

$$\frac{dy}{dx}(6xy^2 - 10y) = -8x - 2y^3$$

$$\frac{dy}{dx} = \frac{-8x - 2y^3}{6xy^2 - 10y}$$

At $x = 1$ and $y = 2$

$$\frac{dy}{dx} = \frac{-8(1) - 2(2)^3}{6[1 \times (2)^2] - 10}$$

$$= \frac{-8 - 16}{24 - 10}$$

$$= \frac{-24}{14}$$

$$dy/dx = -6_{14}$$