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MAT 104 Assignment

Chemical Engineering

- 1 Determine the Stationary point, coordinate of the Stationary point and nature of the Stationary point of the curve

$$y = t^3 - \frac{t^2}{2} - 2t + 4$$

2 If $2y^2 - 5x^4 - 2 - 7y^3 = 0$, Find $\frac{dy}{dx}$

3 Find $\frac{dy}{dx}$, if $4x^2 + 2xy^3 - 5y^2 = 0$ & Evaluate $\frac{dy}{dx}$ when

$$x=1 \text{ and } y=2$$

Solution

1 $y = t^3 - \frac{t^2}{2} - 2t + 4$

i At Stationary point, $\frac{dy}{dx} = 0$

$$\frac{dy}{dx} = 3t^2 - t - 2$$

$$\Rightarrow 3t^2 - t - 2 = 0$$

$$3t^2 - 3t + 2t - 2 = 0$$

$$(3t^2 - 3t) + (2t - 2) = 0$$

$$3t(t-1) + 2(t-1) = 0$$

$$(t-1)(3t+2) = 0$$

$$t-1=0 \quad \& \quad 3t+2=0$$

$$t=1 \quad \& \quad 3t=-2$$

$$t = -\frac{2}{3}$$

\therefore The Stationary point is at $t=1$ or $t=-\frac{2}{3}$

ii The coordinate at $t = 1$

$$y = t^3 - \frac{t^2}{2} - 2t + 4$$

$$y = (1)^3 - \frac{(1)^2}{2} - 2(1) + 4 = \frac{5}{2} = 2.5$$

$$\Rightarrow (1, 2.5)$$

The coordinate @ $t = -\frac{2}{3}$ (0.67)

$$y = t^3 - \frac{t^2}{2} - 2t - 4$$

$$y = \left(-\frac{2}{3}\right)^3 - \frac{\left(-\frac{2}{3}\right)^2}{2} - 2\left(-\frac{2}{3}\right) - 4 = \frac{130}{27} = 4.81$$

$$\Rightarrow (-0.67, 4.81)$$

iii Nature of the Stationary point

$$\frac{dy}{dx} = 3t^2 - t - 2$$

$$\frac{d^2y}{dx^2} = 6t - 1$$

At $t = 1$

$$\frac{d^2y}{dx^2} = 6(1) - 1 = 5$$

At $t = -\frac{2}{3}$

$$\frac{d^2y}{dx^2} = 6\left(-\frac{2}{3}\right) - 1 = -4 - 1 = -5$$

\therefore The stationary point $t = 1$ is a minimum point

The stationary point $t = -\frac{2}{3}$ is a maximum point

$$2 \quad 2y^2 - 5x^4 - 2 - 7y^3 = 0$$

$$4y \frac{dy}{dx} - 20x^3 - 0 - 21y^2 \frac{dy}{dx} = 0$$

$$4y \frac{dy}{dx} - 21y^2 \frac{dy}{dx} - 20x^3 = 0$$

$$(4y - 21y^2) \frac{dy}{dx} = 20x^3$$

$$\frac{dy}{dx} = \frac{20x^3}{(4y - 21y^2)}$$

$$3 \quad 4x^2 + 2xy^3 - 5y^2 = 0$$

$$8x + [2x(3y^2 \frac{dy}{dx}) + (y^3)(2)] - 10y \frac{dy}{dx}$$

$$8x + 6xy^2 \frac{dy}{dx} + 2y^3 - 10y \frac{dy}{dx} = 0$$

$$8x + 2y^3 + 6xy^2 \frac{dy}{dx} - 10y \frac{dy}{dx} = 0$$

$$8x + 2y^3 = 10y \frac{dy}{dx} - 6xy^2 \frac{dy}{dx}$$

$$10y \frac{dy}{dx} - 6xy^2 \frac{dy}{dx} = 8x + 2y^3$$

$$(10y - 6xy^2) \frac{dy}{dx} = 8x + 2y^3$$

$$\frac{dy}{dx} = \frac{8x + 2y^3}{10y - 6xy^2} = \frac{2(4x + y^3)}{2(5y - 3xy^2)}$$

$$\frac{dy}{dx} = \frac{4x + y^3}{y(5 - 3xy)}$$

$$@ \quad x = 1, y = 2$$

$$\frac{dy}{dx} = \frac{4(1) + (2)^3}{2(5 - 3(1)(2))} = \frac{12}{-2} = -6$$