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Question:

1. If the derivative of the following using the first principle

a) $y = \sin\left(\frac{3}{x^2}\right)$

b) $y = 4/x^3$

Solution

a) $y + \Delta y = \sin\theta \quad y = \sin\left(3/x^2\right)$ or $Y = \sin 3x^{-2}$

$y + \Delta y = \sin 3(x+\Delta x)^{-2}$

$\Delta y = \sin 3(x+\Delta x)^{-2} - y$

$\Delta y = \sin 3(x+\Delta x)^{-2} - \sin 3x^{-2}$

$\Delta y = \sin\left(\frac{3}{(x+\Delta x)^2}\right) - \sin 3x^{-2}$ --- (i)

Recall $\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\frac{A-B}{2}$ --- (ii)

Comparing equation (i) & (ii)

$\Delta y = A = \frac{3}{(x+\Delta x)^2}$ and $B = 3x^{-2}$

$\frac{A+B}{2} = \left[\frac{\frac{3}{(x+\Delta x)^2} + 3/x^2}{2}\right]$ ~~$\sin\left(3/(x+\Delta x)^2\right.$~~ $\frac{A-B}{2} = \frac{3/(x+\Delta x)^2 - 3/x^2}{2}$

$\Delta y = 2\cos\left[\frac{3/(x+\Delta x)^2 + 3/x^2}{2}\right]\sin\left(\frac{3/(x+\Delta x)^2 - 3/x^2}{2}\right)$

$= 2\cos\left(\frac{\frac{3x^2 + 3(x+\Delta x)^2}{x^2(x+\Delta x)^2}}{2}\right) \cdot \sin\left(\frac{\frac{3x^2 - 3(x-\Delta x)^2}{x^2(x+\Delta x)^2}}{2}\right)$

$= 2\cos\left(\frac{\frac{6x^2 + 6\Delta x + 3\Delta x^2}{x^2(x+\Delta x)^2}}{2}\right)\sin\left(\frac{\frac{-6\Delta x - 3\Delta x^2}{x^2(x+\Delta x)^2}}{2}\right)$

divide through by Δx

$$\frac{\Delta y}{\Delta x} = \frac{2\cos\left(\frac{6x^2+6\Delta x+3\Delta x^2}{\frac{x^2(x+\Delta x)^2}{2}}\right)\cdot \sin\left(\frac{\frac{-6\Delta x-3\Delta x^2}{x^2(x-\Delta x)^2}}{2}\right)}{\Delta x}$$

multiply the Numerator and Denominator by $\frac{1}{2}$

$$\frac{\Delta y}{\Delta x} = \frac{1}{2}\times\cos\left(\frac{6x^2}{x^2(x+\Delta x)^2}\times\frac{1}{2}+\frac{6\Delta x}{x^2(x+\Delta x)^2}+\frac{1}{2}+\frac{3\Delta x^2}{2(x^2(x+\Delta x)^2)}\right)\cdot$$

$$\frac{\sin\left(\frac{-3\Delta x}{x^2(x+\Delta x)^2}-\frac{3\Delta x^2}{(2x^2(x+\Delta x))}\right)}{\frac{\Delta x}{2}}$$

$$\lim_{\Delta x\to x}\frac{\Delta y}{\Delta x} = \cos\left(\frac{3x^2}{x^2(0+x)^2}+0+0\right),\ \lim_{\Delta x\to 0}\sin(0.0)\times\frac{0}{2}=0$$

$$\frac{dy}{dx} = \cos\left(\frac{3x^2}{x^2 x^2}\right) = \cos\frac{3}{x^2}$$

b) $y = 4/x^2$

Solution

$y = 4/x^2 - y = 4x^{-3}$

$\Delta y + \Delta y = 4(x+\Delta x)^{-3}$

$\Delta y = 4(x+\Delta x)^{-3} - y$

$\Delta y = 4(x+\Delta x)^{-3} - 4x^{-3}$

$$\Delta y = \frac{4}{(x+\Delta x)^3} - \frac{4}{x^3}$$

$$\Delta y = \frac{4}{(3x^2\Delta x+3x\Delta x^3+\Delta x^3+\Delta x^3} - \frac{4}{x^3}$$

$$\Delta y = \frac{4x^3 - 4(3x^2\Delta x+3x\Delta x^2+\Delta x^3+x^3)}{x^3(3x^2\Delta x+3x\Delta x^2+\Delta x^3+x^3)}$$

$$\Delta y = \frac{\cancel{4x^3} - 12x^2\Delta x - 12x\Delta x^2 - 4\Delta x^3 - \cancel{4x^3}}{x^3(3x^2\Delta x + 3x\Delta x^3 + \Delta x^3 + x^3}$$

$$\Delta y = \frac{-12x^2\Delta x - 12\Delta x^2 - 4\cancel{\Delta}x^3}{x^3(3x^2\Delta x + 3x\Delta x^2 + \Delta x^3 + x^3)}$$

$\Delta y =$ Let $t = 3x^3\Delta x + 3x\Delta x^2 + \Delta x^2 + x^3$

$$\therefore \Delta y = \frac{-12\cancel{x^2}dx}{\cancel{x^3}t} - \frac{12x\Delta x^2}{\cancel{x^3}t} - \frac{4dx^3}{\cancel{x^3}t}$$

$$\frac{\Delta y}{\Delta y} - \frac{-17dx}{xt} - \frac{12\Delta x^2}{x^2t} - \frac{4\Delta x^2}{x^1t}$$

multiply both sides by $1/\Delta x$

$$\frac{\Delta y}{\Delta x} = \frac{-12\cancel{\Delta x}}{\cancel{x}t} \times \frac{1}{\cancel{\Delta x}} - \frac{12\cancel{\Delta x}^2}{x^2t} \times \frac{1}{\Delta x} - \frac{4\cancel{\Delta x}^3}{x^3t} \times \frac{1}{\cancel{\Delta x}}$$

$$\frac{\Delta y}{\Delta x} = \frac{-12}{\cancel{x}(3x^2\Delta x + 3x\Delta x^2 + \Delta x^3 + x^3)} - \frac{12\Delta x}{x^2(3x^3\Delta x + 3x\Delta x^2 + \Delta x^3 + x^3)}$$

$$- \frac{4\cancel{\Delta}x^2}{x^3(3x^1\Delta x + 3x\Delta x^2 + \Delta x^3 + x^3)}$$

$$\lim_{\Delta x \to 0} \frac{xy}{\Delta x} = \frac{-12}{x(0+0+0+0x^3)} - 0 - 0$$

$$\lim_{\Delta x \to x} \frac{\Delta y}{\Delta x} = \frac{-12}{x^9} = -12x^{-9}$$

$$\therefore dy/dx = -12x^{-9}$$