IFEANYI - OBI ADAEZE CLARE
19/MH-SOI/193

1) Given: $\int \frac{2x}{(4x^2-1)^{\frac{1}{2}}} dx$

$$= 2\int \frac{x}{\sqrt{(4x^2-1)}} dx$$

$$u = (4x^2-1)^{\frac{1}{2}}$$

$$u^2 = 4x^2 - 1$$

$$u^2 + 1 = 4x^2$$

$$x = \sqrt{\frac{u^2+1}{4}}$$

$$\therefore x = \left(\frac{u^2+1}{4}\right)^{\frac{1}{2}}$$

$$\frac{dx}{du} = \frac{1}{2}\left(\frac{u^2+1}{4}\right)^{-\frac{1}{2}} \cdot \frac{u}{2}$$

$$dx = \frac{u\,du}{4}\left(\frac{u^2+1}{4}\right)^{-\frac{1}{2}}$$

$$2\int \left(\frac{u^2+1}{4}\right)^{\frac{1}{2}} \cdot \frac{1}{u} \cdot \frac{u\,du}{4}\left(\frac{u^2+1}{4}\right)^{-\frac{1}{2}}$$

$$\frac{2}{4}\int \frac{u}{u} du$$

$$\frac{2}{4}\int du = \frac{1}{2}[u] + C$$

$$= \frac{1}{2}\sqrt{4x^2-1} + C$$

$$\int \frac{2x}{4x^2-1} dx = \frac{1}{2}\sqrt{4x^2-1} + C$$

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2) $\int \frac{\sin^{-1} x}{\sqrt{1-x^2}} dx$

$\int \sin^{-1} x \cdot (\sqrt{1-x^2})^{-1} dx$

$u = \sin^{-1} x$

$\frac{du}{dx} = \frac{1}{\sqrt{1-x^2}}$

$du = \frac{dx}{\sqrt{1-x^2}}$

$du = (\sqrt{1^2 - x^2})^{-1} dx$

$\int u \, du$

$\frac{u^2}{2} + C$

$= \frac{(\sin^{-1} x)^2}{2} dx$

$\therefore \int \frac{\sin^{-1} x}{\sqrt{1-x^2}} dx = \frac{(\sin^{-1} x)^2}{2} dx$

3) $\int (\tan x)^6 \sec^2 x \, dx$

Solution

$u = \tan x$

$\frac{du}{dx} = \sec^2 x$

$du = \sec^2 x \, dx$

$\int u^6 \, du$

$\left[\frac{u^7}{7}\right] + C$

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$$= \frac{(\tan x)^7}{7} + c$$

$$\therefore \int (\tan x)^6 \sec^2 x \, dx = \frac{(\tan x)^7}{7} + c$$