

Name: Okuntade Olatunde George

mat 104 practice question

Department: Aeronautical Engineering

matric number: 19/Eng 09/018

Questions:

Integrate the following with respect to their variable

1)  $(11-3x)/(x^2+2x-3)$

Solution

$\int \left( \frac{11-3x}{x^2+2x-3} \right) dx$  - Integrating by partial fraction

$$\frac{11-3x}{x^2+2x-3} = \frac{-11+3x}{(x^2+3x)(-x-3)} = \frac{A}{(x-1)} + \frac{B}{(x+3)}$$

Simplifying  $= \frac{A(x+3) + B(x-1)}{(x-1)(x+3)} = \frac{11-3x}{(x^2+3x)(-x-3)}$

$$\frac{11-3x}{(x-1)(x+3)} = \frac{A(x+3) + B(x-1)}{(x-1)(x+3)}$$

multiplying both sides by  $(x-1)(x+3)$

$$\frac{(x-1)(x+3) \times (11-3x)}{(x-1)(x+3)} = \frac{A(x+3) + B(x-1)}{(x-1)(x+3)} \times (x-1)(x+3)$$

$$11-3x = A(x+3) + B(x-1)$$

At  $x+3=0$ ,  $x=-3$ ,  $B(x-1) = 11-3x$

$$B(-3-1) = 11-3(-3)$$

$$B(-4) = 11+9$$

$$B(-4) = 20$$

$$B = \frac{20}{-4} = -5$$

At  $x-1=0$ ,  $x=1$ ,  $A(x+3) = 11-3x$

$$A(1+3) = 11-3(1)$$

$$A(4) = 8$$

$$A = 2$$



$$= \int \frac{2}{x-1} dx + \int \frac{-5}{x+3} dx = \int \frac{4}{x+1} dx + \int \frac{3}{x+3} dx$$

$$= \int \frac{2}{x-1} dx + \int \frac{-5}{x+3} dx = \int \frac{11-3u}{x^2+2x-3} dx$$

$$\text{Let } u = x-1$$

$$\frac{du}{dx} = 1$$

$$\therefore du = dx$$

$$= 2 \int \frac{du}{u-1}$$

$$= 2 \int \frac{du-1}{u}$$

$$= 2 \ln u$$

$$\text{Let } u = x+3$$

$$\frac{du}{dx} = 1, du = dx$$

$$= -5 \int \frac{du}{u+3}$$

$$= -5 \int \frac{du}{u} = -5 \ln u$$

$$= 2 \ln u - 5 \ln u$$

$$\int \left( \frac{11-3u}{x^2+2x-3} \right) dx = 2 \ln u - 5 \ln u$$

$$= 2 \ln(x-1) - 5 \ln(x+3) + C$$

$$2) \frac{2x^3 - 9x - 35}{(x+5)(x-2)(x+3)}$$

Solution

$$\int \frac{2x^2 - 9x - 35}{(x+5)(x-3)(x+2)}$$

by partial fraction

$$= \frac{A}{x+5} + \frac{B}{x-3} + \frac{C}{x+2} = \frac{A(x-3)(x-2) + B(x+5)(x+3) + C(x+5)(x-2)}{(x+5)(x-3)(x+2)}$$

$$\therefore 2x^2 - 9x - 35 = (2x^2 - 14x) + 5(x-7)$$

$$= 2x(x-7) + 5(x-7)$$

$$= (2x+5)(x-7)$$



Note 1

$$\frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} = \frac{A(x-2)(x-3) + B(x+1)(x+3) + C(x+1)(x-2)}{(x+1)(x-2)(x+3)}$$

Multiplying both sides by  $(x+1)(x-2)(x+3)$ , we have:  
 $2x^2 - 9x - 35 = A(x^2 + 3x - 2x - 6) + B(x^2 + 3x + x + 3) + C(x^2 - 2x + x - 2)$

$$2x^2 - 9x - 35 = A(x^2 + x - 6) + B(x^2 + 4x + 3) + C(x^2 - x - 2)$$

$$2x^2 - 9x - 35 = Ax^2 + Ax - 6A + Bx^2 + 4Bx + 3B + Cx^2 - Cx - 2C$$

$$2x^2 - 9x - 35 = Ax^2 + Bx^2 + Cx^2 + Ax + 4Bx - Cx - 6A + 3B - 2C$$

$$= x^2(A+B+C) + x(A+4B-C) + (-6A+3B-2C)$$

Comparing (i) to  $2x^2 - 9x - 35$  ... (ii)  
 $A+B+C=2, A+4B-C=-9, -6A+3B-2C=0$

$A=2-B-C$ , substituting into the other equations.

We have from (iii)  $(2-B-C) + 4B - C = -9$

$$= 2 - B - C + 4B - C = -9$$

$$2 + 3B - 2C = -9, 3B - 2C = -11$$

$$3B - 2C = -11 \quad \text{--- (iv)}$$

We have from (iv)  $-6A + 3B - 2C = 0$

$$= -6(2-B-C) + 3B - 2C = 0$$

$$= -12 + 6B + 6C + 3B - 2C = 0$$

$$= -12 + 9B + 4C = 0$$

$$9B + 4C = 12 \quad \text{--- (v)}$$

From eqn (v) from (v)

$$3B - 2C = -11, \frac{3B}{3} = \frac{-11 + 2C}{3}$$

$$B = \frac{-11 + 2C}{3} \quad \text{--- (vi)}$$

Put (vi) in (v) i.e.  $9B + 4C = 12$

$$3\left(\frac{-11 + 2C}{3}\right) + 4C = 12$$