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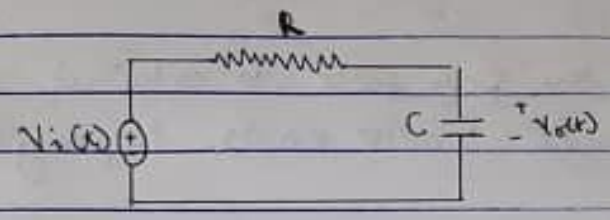
17/ENG04/057
ELECTRICAL ELECTRONICS ENGINEERING
CIRCUIT THEORY (EEE 322)

QUESTION:



Determine the type of filter shown and show that its cut-off frequency is $\omega_c = 1/RC$

Solution:

1. The type of filter shown above is a LOW PASS RC FILTER.

This type of filter is generated when the output of RC is removed from capacitor.

2. Show that $\omega_c = \frac{1}{RC}$;

From the circuit above;

* using the voltage division rule;
  * Converting the components from time domain to frequency domain;

$\therefore$ $R(t) = R(s)$
$C(t) = 1/sC$

* using voltage division rule;

* Note: Transfer function $H(\omega) = \frac{V_o}{V_i}$

The voltage across the capacitor $1/sC$, V_o

$$\therefore H(\omega) = \frac{V_o}{} = \left(\frac{1/sC}{R + 1/sC}\right) * V_i$$

$$H(\omega) = \frac{V_o}{V_i} = \frac{1/sC}{R + 1/sC}$$ * (multiply this by sC)

$$H(\omega) = \frac{V_o}{V_i} = \frac{1}{RsC + 1}$$

Where $s = j\omega$

$$\therefore H(\omega) = \frac{V_o}{V_i} = \frac{1}{j\omega RC + 1}$$

Noting that;

$$H(0) = 1$$
$$H(\infty) = 0$$

$\therefore$ The Cutoff frequency (ω_c) of a low pass filter is obtained by setting the magnitude of $H(\omega)$ to $1/\sqrt{2}$. In other words, frequency above $1/\sqrt{2}$ are be eliminated;

$$\therefore |H(\omega)| = \left| \frac{1}{j\omega RC + 1} \right|$$

$$\frac{1}{\sqrt{2}} = \frac{\sqrt{(1)^2}}{\sqrt{1^2 + (j\omega RC)^2}}$$

$$\therefore \sqrt{2} = \sqrt{1^2 + (j\omega RC)^2}$$

$$\therefore \sqrt{2} = \sqrt{1 + \omega^2 R^2 C^2}$$

Finding the squares of both sides;

$$\therefore (\sqrt{2})^2 = \left(\sqrt{1^2 + (j\omega^2 R^2 C^2)}\right)^2$$

$$\therefore 2 = 1 + (\omega^2 * R^2 * C^2)$$

$$2 - 1 = \omega^2 * R^2 * C^2$$

$$1 = \omega^2 R^2 C^2$$

$$\omega^2 = \frac{1}{R^2 C^2}$$

$\therefore$ Square root all sides;

$$\omega_c = \frac{1}{RC}$$ Q.E.D

$\therefore$ The Cutoff frequency $\omega_c = \frac{1}{R_c}$; also called rolloff frequency.

Representing the Cut-off frequency Graphically on a low pass filter graph.