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Assignment:

b) $y = \sin(3/x^2)$ from first principle

$y + \Delta y = \sin\left[\frac{3}{(x+\Delta x)^2}\right]$

$\Delta y = \sin\left[\frac{3}{(x+\Delta x)^2}\right] - y$

$\Delta y = \sin\left[\frac{3}{(x+\Delta x)^2}\right] - \sin\left[\frac{3}{x^2}\right]$ — ①

Recall $\sin A - \sin B = 2\cos\frac{(A+B)}{2}\sin\frac{(A-B)}{2}$ — ②

Comparing equation ① & ②:

$A = \frac{3}{(x+\Delta x)^2}$ and $B = \left[\frac{3}{x^2}\right]$

$\therefore \frac{A+B}{2} = \frac{3}{(x+\Delta x)^2} + \frac{3}{x^2} =$

$\frac{3x^2 + 3(x+\Delta x)^2}{(x+\Delta x)^2 (x^2)} \div 2$

$= \frac{3x^2 + 3(x+\Delta x)(x+\Delta x)}{(x+\Delta x)^2 (x^2)} \div 2$

$= \frac{3x^2 + 3x + 3\Delta x(x+\Delta x)}{(x+\Delta x)^2 (x^2)} \div 2$

$= \frac{3x^2 + 3x^2 + 3x\Delta x + 3x\Delta x + 3(\Delta x)^2}{(x+\Delta x)^2 (x^2)} \div 2$

$= \frac{9x^2 + 9x\Delta x + 3(\Delta x)^2}{(x+\Delta x)^2 (x^2)} \times \frac{1}{2}$

$= \frac{9x^2 + 9x\Delta x + 3(\Delta x)^2}{2[(x+\Delta x)^2 (x^2)]}$ //

And $\frac{A-B}{2} = \frac{\left[\frac{3}{(x+\Delta x)^2}\right] - \left[\frac{3}{x^2}\right]}{2}$

$\therefore \frac{A-B}{2} = \frac{3}{(x+\Delta x)^2} - \frac{3}{x^2} =$

$\frac{3x^2 - 3(x+\Delta x)^2}{(x+\Delta x)^2 (x^2)} \div 2$

$= \frac{3x^2 - 3(x+\Delta x)(x+\Delta x)}{(x+\Delta x)^2 (x^2)} \div 2$

$= \frac{3x^2 - (3x + 3\Delta x)(x+\Delta x)}{(x+\Delta x)^2 (x^2)} \div 2$

$= \frac{3x^2 - (3x^2 + 3x\Delta x + 3x\Delta x + 3(\Delta x)^2)}{(x+\Delta x)^2 (x^2)} \div 2$

$= \frac{3x^2 - (3x^2 + 9x\Delta x + 3(\Delta x)^2)}{(x+\Delta x)^2 (x^2)} \div 2$

$= \frac{3x^2 - 3x^2 - 9x\Delta x - 3(\Delta x)^2}{(x+\Delta x)^2 (x^2)} \div 2$

$= \frac{-9x\Delta x - 3(\Delta x)^2}{2[(x+\Delta x)^2 (x^2)]}$

Hence $\sin\left[\frac{3}{(x+\Delta x)^2}\right] - \sin\left[\frac{3}{x^2}\right] =$

$2\cos\left[\frac{9x^2 + 9x\Delta x + 3(\Delta x)^2}{2[(x+\Delta x)^2 (x^2)]}\right] \sin\left[\frac{-9x\Delta x - 3(\Delta x)^2}{2[(x+\Delta x)^2 (x^2)]}\right]$

Divide through by Δx:

$\frac{\Delta y}{\Delta x} = 2\cos\left[\frac{9x^2 + 9x\Delta x + 3(\Delta x)^2}{2[(x+\Delta x)^2 (x^2)]}\right]$

$$\frac{\Delta y}{\Delta x} = 2\cos\left[\frac{9x^2+9x\Delta x+3(\Delta x)^2}{2(x+\Delta x)^2(x^3)}\right]\sin\left[\frac{-9x\Delta x-3(\Delta x)^2}{2(x+\Delta x)^2(x^3)}\right]\times\frac{1}{2}$$

$$\lim_{\Delta x\to 0}\cos\left[\frac{9x^2+0+0}{2x^4}\right]\sin\left[\frac{0}{2x^4}\right]\frac{x}{2}$$

Hence, $\frac{dy}{dx} = \cos\frac{9x^2}{2x^4}$ //

b) $y = \frac{4}{x^3}$ from first principle

$$y+\Delta y = \frac{4}{(x+\Delta x)^3}$$

$$y+\Delta y = \frac{4}{x^3+3x^2\Delta x+3x(\Delta x)^2+\Delta x^3}$$

$$\Delta y = \frac{4}{x^3+3x^2\Delta x+3x(\Delta x)^2+\Delta x^3} - y$$

$$\Delta y = \frac{4}{x^3+3x^2\Delta x+3x(\Delta x)^2+\Delta x^3} - \frac{4}{x^3}$$

$$\Delta y = \frac{4x^3 - 4(x^3+3x^2\Delta x+3x(\Delta x)^2+\Delta x^3)}{(x^3+3x^2\Delta x+3x(\Delta x)^2+\Delta x^3)x^3}$$

$$\Delta y = \frac{4x^3-(x^3+3x^2\Delta x+3x(\Delta x)^2+\Delta x^3)}{(x^3+3x^2\Delta x+3x(\Delta x)^2+\Delta x^3)x^3}$$

$$\Delta y = \frac{4x^3-x^3-3x^2\Delta x-3x(\Delta x)^2-\Delta x^3}{(x^3+3x^2\Delta x+3x(\Delta x)^2+\Delta x^3)x^3}$$

Divide both sides by Δx

$$\frac{\Delta y}{\Delta x} = \frac{4(-3x^2\Delta x-3x(\Delta x)^2-\Delta x^3)\div\Delta x}{(x^3+3x^2\Delta x+3x(\Delta x)^2+\Delta x^3)x^3\quad 1}$$

$$\frac{\Delta y}{\Delta x} = \frac{4(-3x^2\Delta x-3x(\Delta x)^2-\Delta x^3)\times 1}{(x^3+3x^2\Delta x+3x(\Delta x)^2+\Delta x^3)x^3\quad \Delta x}$$

$$\frac{dy}{dx} = \frac{4(-3x^2-3x\Delta x-\Delta x^2)\Delta x\times 1}{(x^3+3x^2\Delta x+3x(\Delta x)^2+\Delta x^3)x^3}$$

$$\frac{\Delta y}{\Delta x} = \frac{4(-3x^2-3x\Delta x-\Delta x^2)}{(x^3+3x^2\Delta x+3x(\Delta x)^2+\Delta x^3)x^3}$$

$\lim_{\Delta x\to 0}$

$$\frac{\Delta y}{\Delta x} = \frac{4(-3x^2)}{(x^3+0)x^3}$$

$$\frac{\Delta y}{\Delta x} = \frac{-12x^2}{x^4} = \frac{-12\times x\times x}{x\times x\times x\times x}$$

$$= \frac{-12}{x^2}$$ //

2) Find the integrals of the following:

a) $\int \frac{dx}{(x^2+36)}$

Let $u=(x^2+36)$

$\frac{du}{dx} = 2x$

$dx = \frac{du}{2x}$

$\int \frac{dx}{(x^2+36)} = \int \frac{1}{u}\left(\frac{du}{2x}\right) =$

$\frac{1}{2x}\int \frac{du}{u}$

$= \frac{1}{2x}\left[\frac{u^2}{2}\right] + 2$

$= \frac{1}{4x}\left\{u^2\right] + 2$

$= \frac{1}{4x}(x^2+36)^2 + C$ //

2) $\int \frac{dx}{(x^2+13)}$

Let $u = (x^2+13)$

$\frac{du}{dx} = 2x$

$dx = \frac{du}{2x}$

$\int \frac{dx}{x^2+13} = \int \frac{1}{u}\left(\frac{du}{2x}\right)$

$= \frac{1}{2x}\int \frac{du}{u}$

$= \frac{1}{2x}\left[\frac{u^2}{2}\right] + 2$

$= \frac{1}{4x}\left\{u^2\right] + 2$

$= \frac{1}{4x}(x^2+13)^2 + C$ //