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MAT 102 MECHATRONICS ENGINEERING
19/ENG05/014

1a $y = \sin\frac{3}{x^2}$

$y + \Delta y = \sin\frac{3}{(x+\Delta x)^2}$

$\Delta y = \sin\frac{3}{(x+\Delta x)^2} - \sin\frac{3}{x^2}$ $\quad \sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\cdot\sin\left(\frac{A-B}{2}\right)$

$\Delta y = 2\cos\frac{\frac{3}{(x+\Delta x)^2} + \frac{3}{x^2}}{2} \sin\frac{\frac{3}{(x+\Delta x)^2} - \frac{3}{x^2}}{2}$

$\Delta y = 2\cos\frac{3x^2 + 3(x+\Delta x)^2}{2x^2(x+\Delta x)^2} \sin\frac{3x^2 - 3(x+\Delta x)^2}{2x^2(x+\Delta x)^2}$

$\frac{\Delta y}{\Delta x} = \frac{2\cos\frac{3x^2+3(x+\Delta x)^2}{2x^2(x+\Delta x)^2} \sin\frac{3x^2 - 3(x+\Delta x)^2}{2x^2(x+\Delta x)^2}}{\Delta x}$

$\frac{\Delta y}{\Delta x} = 2\cos\frac{3x^2+3(x+\Delta x)^2}{2x^2(x+\Delta x)^2} \sin\frac{-6x\Delta x - 3(\Delta x)^2}{2x^2(x+\Delta x)^2}$

$\frac{\Delta y}{\Delta x} = 2\cos\frac{3x^2+3(x+\Delta x)^2}{2x^2(x+\Delta x)^2} \frac{\sin\frac{-6x\Delta x - 3(\Delta x)^2}{2x^2(x+\Delta x)^2}}{\Delta x\left[\frac{-6x-3(\Delta x)}{2x^2(x+\Delta x)^2}\right]} \times \frac{-6x-3(\Delta x)}{2x^2(x+\Delta x)^2}$

$\frac{\Delta y}{\Delta x} = 2\cos\frac{3x^2+3(x+\Delta x)^2}{2x^2(x+\Delta x)^2} \frac{\sin\frac{-6x\Delta x - 3(\Delta x)^2}{2x^2(x+\Delta x)^2}}{\frac{-6x\Delta x - 3\Delta x^2}{2x^2\, x+\Delta x}} \times \frac{-6x-3\Delta x}{2x^2(x+\Delta x)^2}$

$\lim_{\Delta x \to 0}\frac{\Delta y}{\Delta x} = 2\cos\frac{3x^2+3(x+\Delta x)^2}{2x^2(x+\Delta x)^2} \times \frac{-6x-3\Delta x}{2x^2(x+\Delta x)^2}$

$\frac{dy}{dx} = \cos\frac{3x^2+3(x+0)^2}{2x^2(x+0)^2} \times \frac{-6x-3(0)}{2x^2(x+0)^2}$

$= \cos\frac{3x^2+3x^2}{2x^4} \; \frac{-6x}{x^4}$

$= \frac{-6x}{x^4}\cos\frac{6x^2}{2x^4}$

$\frac{dy}{dx} = \frac{-6}{x^3}\cos\frac{3}{x^2}$

1b $y = \frac{4}{x^3}$

$\Delta y + y = \frac{4}{(x+\Delta x)^3}$

$\Delta y = \frac{4}{(x+\Delta x)^3} - y$

$\Delta y = \frac{4}{(x+\Delta x)^3} - \frac{4}{x^3}$

$\Delta y = \frac{4x^3 - 4(x+\Delta x)^3}{x^3(x+\Delta x)^3}$

$\Delta y = \frac{4x^3 - (4x^3 + 12x^2\Delta x + 12x(\Delta x)^2 + 4(\Delta x)^3}{x^3(x+\Delta x)^3}$

$\Delta y = \frac{-12x^2\Delta x - 12x(\Delta x)^2 - 4(\Delta x)^3}{x^3(x+\Delta x)^3}$

$\frac{\Delta y}{\Delta x} = \frac{(-12x^2 - 12x\Delta x - 4(\Delta x)^2)\Delta x}{x^3(x+\Delta x)^3} \times \frac{1}{\Delta x}$

$\frac{\Delta y}{\Delta x} = \frac{-12x^2 - 12x\Delta x - 4(\Delta x)^2}{x^3(x+\Delta x)^3}$

$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{-12x^2 - 12x(0) - 4(0)^2}{x^3(x+0)^3}$

$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{-12x^2}{x^6}$

$\frac{dy}{dx} = \frac{-12}{x^4}$

2a $\int \frac{dx}{x^2+36}$

$\int \frac{1}{x^2+6^2} dx$

$x = 6\tan\theta$

$\frac{dx}{d\theta} = 6\sec^2\theta \qquad dx = 6\sec^2\theta \, d\theta$

$\int \frac{6\sec^2\theta \, d\theta}{6^2\tan^2\theta + 6^2}$

$\int \frac{6\sec^2\theta \, d\theta}{6^2(1+\tan^2\theta)} \qquad 1+\tan^2\theta = \sec^2\theta$

$\int \frac{1}{6} \frac{\sec^2\theta \, d\theta}{\sec^2\theta}$

$\int \frac{1}{6} \, d\theta$

$\frac{1}{6}[\theta] + C$

$= \frac{1}{6}\tan^{-1}\frac{x}{6} + C$

2b) $\int \frac{1}{x^2+13} \, dx$

$\int \frac{1}{x^2+(\sqrt{13})^2} \, dx$

$x = \sqrt{13}\tan\theta$

$\frac{dx}{d\theta} = \sqrt{13}\sec^2\theta \qquad dx = \sqrt{13}\sec^2 \, d\theta$

$\int \frac{\sqrt{13}\sec^2\theta \, d\theta}{(\sqrt{13})^2\tan^2\theta + (\sqrt{13})^2}$

$\int \frac{\sqrt{13}\sec^2\theta \, d\theta}{(\sqrt{13})^2(1+\tan^2\theta)} \qquad 1+\tan^2\theta = \sec^2\theta$

$\int \frac{1}{\sqrt{13}} \frac{\sec^2\theta \, d\theta}{\sec^2\theta}$

$\int \frac{1}{\sqrt{13}} \, d\theta$

$$\frac{1}{\sqrt{13}}[\theta] + C$$

$$\frac{\sqrt{13}}{13} \tan^{-1} \frac{\sqrt{13}}{13} x + C$$