

$$1) \quad y = \sin\left(\frac{3}{x^2}\right)$$

$$y + \Delta y = \sin\left(\frac{3}{x^2} + \Delta x^2\right)$$

$$\Delta y = \sin\left(\frac{3}{x^2} + \Delta x^2\right) - y$$

$$\Delta y = \sin\left(\frac{3}{x^2} + \Delta x^2\right) - \sin\left(\frac{3}{x^2}\right)$$

from the factor formulae.

$$\Delta y = \sin\left(\frac{3}{x^2} + \Delta x^2\right) - \sin\left(\frac{3}{x^2}\right)$$

$$= \Delta y = 2 \cos\left(\frac{\frac{3}{x^2} + \frac{\Delta x^2}{2}}{2}\right) \sin \frac{\Delta x^2}{2}$$

Divide by Δx^2

$$\frac{\Delta y}{\Delta x} = 2 \cos\left(\frac{\frac{3}{x^2} + \frac{\Delta x^2}{2}}{2}\right) \sin \frac{\Delta x^2}{2}$$

Δx^2

Multiply denominator by $y =$

$$\frac{\Delta y}{\Delta x} = \frac{2 \cos\left(\frac{3}{x^2} + \frac{\Delta x^2}{2}\right) \sin\left[\frac{\Delta x^2}{2}\right] \times \frac{1}{2}}{\Delta x^2 \times \frac{1}{2}}$$

$$\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = 2 \cos\left(\frac{3}{x^2} + 0\right) \times 1$$

Since $\lim_{\Delta x \rightarrow 0} \frac{\sin(\Delta x^2/2)}{\Delta x^2/2} = 1$

Hence $\frac{dy}{dx} = 2 \cos 3/x^2$

$$2a) \int \frac{dx}{x^2 + 36}$$

$$\int \frac{dx}{x^2 + 36}$$

$$\int \frac{dx}{x^2 + 36} = \int \frac{dx}{x^2 + 6^2}$$

$$x = 6 \tan \theta$$

$$\frac{dx}{d\theta} = 6 \sec^2 \theta$$

$$d\theta$$

$$dx = 6 \sec^2 \theta d\theta$$

$$x^2 + 6^2 = 6 \tan^2 \theta + 6^2 = 6^2 (\tan^2 \theta + 1)$$

$$= 36 \sec^2 \theta$$

$$\int \frac{6 \sec^2 \theta d\theta}{36 \sec^2 \theta} = \int \frac{d\theta}{6} = \frac{1}{6} \int d\theta$$

$$\frac{1}{6} [\theta] + C$$

$$6$$

$$= \frac{1}{6} \tan^{-1} x/6 + C$$

$$2b) \int \frac{dx}{x^2 + 13}$$

$$\int \frac{dx}{(x^2 + 13)} = \int \frac{dx}{x^2 + 4 + 9}$$

$$\int \frac{dx}{(x+2)^2 + 3^2}$$

$$(x+2) = 3 \tan \theta$$

$$x = 3 \tan \theta - 2$$

$$dx/d\theta = 3 \sec^2 \theta \Rightarrow dx = 3 \sec^2 \theta d\theta$$

$$(x+2)^2 + 3^2 = 3^2 \tan^2 \theta + 3^2 =$$

$$= 3^2 \sec^2 \theta d\theta$$

$$\int \frac{3 \sec^2 \theta d\theta}{3^2 \sec^2 \theta} = \frac{1}{3} \int d\theta$$

$$= \frac{1}{3} [\theta] + C$$

$$= \frac{1}{3} \tan^{-1} \left(\frac{x+2}{3} \right) + C$$

$$b) y = 4/x^3$$

$$y = 4x^{-3} \quad \text{und in die}$$

$$(4)(-3)x^{-3-1}$$

$$= -12x^4 = \frac{-12}{x^4}$$