Usman Zainab Damilola
19/MHS01/426
Mat104 Assignment

USMAN ZAINAB DAMILOLA
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MEDICINE AND SURGERY
MAT 104

1. $\int \frac{2x}{\sqrt{4x^2-1}} dx$

Let $u = 4x^2 - 1$

$du/dx = 8x$

$dx = \frac{du}{8x} = \frac{1}{8x} du$

$\int \frac{2x}{\sqrt{4x^2-1}} dx = \int \frac{2x}{\sqrt{u}} \cdot \frac{1}{8x} du$

$\int \frac{2x}{\sqrt{4x^2-1}} dx = \int \frac{1}{4} \cdot \frac{1}{\sqrt{u}} du$

$\int \frac{2x}{\sqrt{4x^2-1}} dx = \frac{1}{4} \int \frac{1}{u^{1/2}} du$

$\int \frac{2x}{\sqrt{4x^2-1}} dx = \frac{1}{4} \cdot \int u^{-1/2} du$

$\int \frac{2x}{\sqrt{4x^2-1}} dx = 1/4 \left(\frac{u^{-1/2+1}}{-1/2+1} \right)$

$\int \frac{2x}{\sqrt{4x^2-1}} dx = 1/4 \left(\frac{u^{1/2}}{1/2} \right)$

$\cancel{1/4 \left(u^{1/2} \times 1/2 \right)}$

$\int \frac{2x}{\sqrt{4x^2-1}} dx = 1/4 \left(u^{1/2} \div 1/2 \right)$

$1/4 \left(u^{1/2} \times 2 \right)$

$\int \frac{2x}{\sqrt{4x^2-1}} dx = 2/4 \left(u^{1/2} \right)$

$1/2 \cdot u^{1/2}$

But $u = 4x^2 - 1$

$\int \frac{2x}{\sqrt{4x^2-1}} dx = 1/2 \cdot (4x^2-1)^{1/2}$

$1/2 \sqrt{4x^2-1} + C$

$\therefore \int \frac{2x}{\sqrt{4x^2-1}} dx = \frac{1}{2} \sqrt{4x^2-1} + C$

2) $\int \frac{\sin^{-1}x}{\sqrt{1-x^2}}\,dx = \frac{(\sin^{-1}x)^2}{2} + c$

$\int \frac{\sin^{-1}x}{\sqrt{1-x^2}}\,dx = \int \frac{1}{\sqrt{1-x^2}} \cdot \sin^{-1}x$

Let $u = \sin^{-1}x$

$u = \frac{x}{\sin}$

$x = \sin u \quad dx/du = \cos u$

Recall

$\sin^2 u + \cos^2 u = 1$

$\cos^2 u = 1 - \sin^2 u$

But $x = \sin u$

$\therefore \cos u = \sqrt{1-x^2}$

$dx/du = \cos u = \sqrt{1-x^2}$

$\therefore du/dx = \frac{1}{\sqrt{1-x^2}} \qquad \therefore dx = du \cdot \sqrt{1-x^2}$

$\int \frac{\sin^{-1}x}{\sqrt{1-x^2}}\,dx = \int \frac{u}{\cancel{\sqrt{1-x^2}}} \cdot du \cdot \cancel{\sqrt{1-x^2}}$

$\int \frac{\sin^{-1}x}{\sqrt{1-x^2}}\,dx = \int u\,du$

$\int \frac{\sin^{-1}x}{\sqrt{1-x^2}}\,dx = \frac{u^{1+1}}{1+1}$

$\int \frac{\sin^{-1}x}{\sqrt{1-x^2}}\,dx = \frac{u^2}{2}$

But $u = \sin^{-1}x$

$\int \frac{\sin^{-1}x}{\sqrt{1-x^2}}\,dx = \frac{(\sin^{-1}x)^2}{2} + c$

$\therefore \int \frac{\sin^{-1}x}{\sqrt{1-x^2}}\,dx = \frac{(\sin^{-1}x)^2}{2} + c$

3. $\int (\tan x)^6 \sec^2 x \, dx = \frac{(\tan x)^7}{7} + C$

Sol

Let $u = \tan x$

$\therefore du/dx = \sec^2 x$

$dx = \frac{du}{\sec^2 x} = \frac{1}{\sec^2 x} du$

$\therefore \int (\tan x)^6 \sec^2 x \, dx = \int u^6 \cancel{\sec^2 x} \cdot \frac{1}{\cancel{\sec^2 x}} du$

$\int (\tan x)^6 \sec^2 x \, dx = \int u^6 \, du$

$\int (\tan x)^6 \sec^2 x \, dx = \frac{u^{6+1}}{6+1}$

$\int (\tan x)^6 \sec^2 x \, dx = \frac{u^7}{7}$

But $u = \tan x$

$\int (\tan x)^6 \sec^2 x \, dx = \frac{(\tan x)^7}{7}$

$\therefore \int (\tan x)^6 \sec^2 x \, dx = \frac{(\tan x)^7}{7} + C$