19/ENG05/019

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MECHATRONICS

1 Find dy/dx of $y = \sin\frac{3}{x^2}$ from first principle.

Solution

If $y = \sin\left(\frac{3}{x^2}\right) = f(x)$ let $h = \Delta x$

$$\frac{dy}{dx} = f'(x) = \lim_{h\to 0} = \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{\sin\frac{3}{(x+h)^2} - \sin 3/x^2}{h}$$

If $\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\cdot\sin\left(\frac{A-B}{2}\right)$

$$= \lim_{h\to 0} 2\cos\left[\frac{\frac{3}{(x+h)^2} + 3/x^2}{2}\right]\cdot\sin\left[\frac{3/(x+h)^2 + 3/x^2}{2}\right]$$

$$\frac{\lim_{h\to 0} 2\cos\left[\frac{\frac{3}{(x+h)^2} + \frac{3}{x^2}}{2}\right]\cdot\lim_{h\to 0}\sin\left[\frac{\frac{3x^2-3(x+h)^2}{x^2(x+h)^2}}{2}\right]}{h}$$

$$\lim_{h} 2\cos\left(\frac{3/x^2 + 3/x^2}{2}\right)\lim \sin\frac{\left[\frac{\frac{3x^2-3x^2-6xh-3h^2}{x^2(x+h)^2}}{2}\right]}{h}$$

$$2\cos 3/x^2 \times \lim \sin\frac{\frac{-6xh-3h^2}{2x^2(x+h)^2}}{h}$$

$$\frac{\sin ah}{a} = a$$

1a contd

$$2\cos\left(\frac{3}{x^2}\right) \times \lim \frac{-6x-3h}{2x^2(x+h)^2}$$

$$2\cos\left(3/x^2\right) \times \frac{-6x}{2(x^2 \times x^2)}$$

$$2\cos\left(3/x^2\right) \times \frac{-6x}{2x^4}$$

$$= \frac{-6x}{x^4}\cos\left(\frac{3}{x^2}\right)$$

$$= -6/x^3 \cos\left(3/x^2\right)$$

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B) $y = \frac{4}{x^3}$

$$\frac{dy}{dx}\left(\frac{4}{x^3}\right)$$

Using first principle

$$y + \Delta y = \frac{4}{(x+\Delta x)^3}$$

$$\Delta y = \frac{4}{(x+\Delta x)^3} - \frac{4}{x^3}$$

$$\frac{\Delta y}{\Delta x} = \frac{\frac{4}{(x+\Delta x)^3} - \frac{4}{x^3}}{\Delta x}$$

$$\frac{\Delta y}{\Delta x} = \frac{\frac{4x^3 - 4(x+\Delta x)^3}{x^3(x+\Delta x)^3}}{\Delta x}$$

$$\frac{\Delta y}{\Delta x} = \frac{\frac{4x^3 - 4(x^3 + 3x^2\Delta x + 3x\Delta x^2 + \Delta x^3)}{x^3(x+\Delta x)^3}}{\Delta x}$$

$$= \frac{\frac{4x^3 - 4x^3 - 12x^2\Delta x - 12x\Delta x^2 - 4\Delta x^3}{x^3(x+\Delta x)^3}}{\Delta x}$$

$$\frac{\dfrac{-12x^2\Delta x - 12x\Delta x^2 - 4\Delta x^3}{x^3(x+\Delta x)^3}}{\Delta x}$$

$$\frac{\cancel{\Delta x}(-12x^2 - 12x\Delta x - 4\Delta x^2)}{x^3(x+\Delta x)^3} \times \frac{1}{\cancel{\Delta x}}$$

$$= \frac{-12x^2 - 12x\Delta x - 4\Delta x^2}{x^3(x+\Delta x)^3}$$

$$\lim_{\Delta x \to 0} \frac{-12x^2 - 12x(0) - 4(0)^2}{x^3(x+0)^3}$$

$$= \frac{-12x^2}{x^6}$$

$$= \frac{-12}{x^4} = -12x^{-4}$$

2 $\displaystyle\int \frac{dx}{x^2+36}$

$$= \int \frac{dx}{x^2+6^2}$$

$x = 6\tan\theta$ $\qquad \tan\theta = \frac{x}{6} \quad \theta = \tan^{-1}\frac{x}{6}$

$\frac{dx}{d\theta} = 6\sec^2\theta$

$dx = 6\sec^2\theta \, d\theta$

$x^2 + 6^2 = 6^2\tan^2\theta + 6^2 = 6^2(\tan^2\theta + 1)$

$= 36(\tan^2\theta + 1)$

$= 36\sec^2\theta$

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$$\Rightarrow \quad \frac{\cancel{6}\sec^2\theta\, d\theta}{\cancel{36}_{6}\cancel{\sec^2\theta}} = \int \frac{d\theta}{6} = \frac{1}{6}\int d\theta$$

$$= \frac{1}{6}[\theta] + c \qquad \theta = \tan^{-1}\frac{x}{6}$$

$$= \frac{1}{6}\tan^{-1}\frac{x}{6} + c$$

b $\displaystyle\int \frac{dx}{x^2+13}$

$$\tan\theta = \frac{x}{\sqrt{13}}$$

$$\theta = \tan^{-1}\frac{x}{\sqrt{13}}$$

$x = \sqrt{13}\tan\theta$

$\frac{dx}{d\theta} = \sqrt{13}\sec^2\theta$

$dx = \sqrt{13}\sec^2\theta\, d\theta$

$$x^2 + (\sqrt{13})^2 = (\sqrt{13})^2\tan^2\theta + (\sqrt{13})^2 = (\sqrt{13})^2(\tan^2\theta + 1)$$

$$= (\sqrt{13})^2\sec^2\theta \quad = 13\sec^2\theta$$

$$\Rightarrow \quad \frac{\sqrt{13}\,\cancel{\sec^2\theta}\, d\theta}{13\,\cancel{\sec^2\theta}} = \int \frac{\sqrt{13}}{13}\, d\theta = \frac{\sqrt{13}}{13}\int d\theta$$

$$= \frac{\sqrt{13}}{13}[\theta] + c$$

$$= \frac{\sqrt{13}}{13}\tan\frac{x}{\sqrt{13}} + c$$