① $\int \frac{2x}{\sqrt{4x^2-1}}\,dx$

let $u = \sqrt{4x^2-1} = (4x^2-1)^{1/2}$

$\frac{du}{dx} = \frac{1}{2}(4x^2-1)^{-1/2}\cdot 8x$

$\frac{du}{dx} = 4x(4x^2-1)^{-1/2}$

$dx = \frac{du}{4x(4x^2-1)^{-1/2}} = \frac{(4x^2-1)^{1/2}}{4x}\,du$

we have

$2\int \frac{x}{u}\,dx = 2\int \frac{x}{(4x^2-1)^{1/2}}\;\frac{(4x^2-1)^{1/2}}{4x}\,du$

$= \frac{1}{2}\int du$

$= \frac{1}{2}u + c = \frac{1}{2}\sqrt{4x^2-1} + c$

② $\int \frac{\sin^{-1}x}{\sqrt{1-x^2}}\,dx$

$= \int \sin^{-1}x\,(1-x^2)^{-1/2}\,dx$

let $u = \sin^{-1}x$

$du = (1-x^2)^{-1/2}\,dx$

$\int u\,du = \frac{u^2}{2} + c$

$= \frac{(\sin^{-1}x)^2}{2} + c$

③ $\int (\tan x)^6 \sec^2 x\,dx$

let $u = \tan x$

$du = \sec^2 x\,dx$

we have

$\int u^6\,du = \frac{u^7}{7} + c$

$= \frac{(\tan x)^7}{7} + c$