

Alisdair Friday Johnny

Civil Engineering

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MAT 104 Assignment

Answers

$$1.) y = t^3 - \frac{t^2}{2} - 2t - 4$$

$$\frac{dy}{dt} = 3t^2 - t - 2$$

At stationary point, $\frac{dy}{dx} = 0$

$$\therefore 3t^2 - t - 2 = 0$$

Using factorization method

$$3t^2 - t - 2 = 0$$

$$3t^2 - 3t + 2t - 2 = 0$$

$$3t(t-1) + 2(t-1) = 0$$

$$(3t-2)(t-1) = 0$$

$$\therefore t = \frac{2}{3} \text{ and } t = 1$$

The stationary points are $\frac{2}{3}$ and 1

$$\text{At } t = \frac{2}{3}$$

$$y = \left(\frac{2}{3}\right)^3 - \frac{\left(\frac{2}{3}\right)^2}{2} - 2\left(\frac{2}{3}\right) - 4$$

$$\therefore y = -5.26$$

$$\text{At } t = 1$$

$$y = (1)^3 - \frac{(1)^2}{2} - 2(1) - 4$$

$$\therefore y = -5.5$$

\therefore The coordinates are $(\frac{2}{3}, -5.26)$ and $(1, -5.5)$

Nature of stationary point

$$\frac{dy}{dt} = 3t^2 - t - 2$$

$$\frac{d^2y}{dt^2} = 6t - 1 \quad \therefore$$

$$\text{At } t = 2/3$$

$$\frac{d^2y}{dt^2} = 4 - 1 \Rightarrow 3$$

$$\text{At } t = 1$$

$$\frac{d^2y}{dt^2} = 6 - 1 \Rightarrow 5$$

\therefore At $(2/3, -5.26)$, there is a minimum point and at $(1, -5.5)$, there is also a minimum point.

$$2.) \quad 2y^2 - 5x^4 - 2 - 7y^3 = 0$$

$$4y \frac{dy}{dx} - 20x^3 - 21y^2 \frac{dy}{dx} = 0$$

$$4y \frac{dy}{dx} - 21y^2 \frac{dy}{dx} = 20x^3$$

$$\frac{dy}{dx} (4y - 21y^2) = 20x^3$$

$$\therefore \frac{dy}{dx} = \frac{20x^3}{4y - 21y^2}$$

$$3.) 4x^2 + 2xy^3 - 5y^2 = 0$$

$$8x + \left(\frac{2x \cdot 3y^2 dy}{dx} + y^3 \cdot 2 \right) - 10y \frac{dy}{dx} = 0$$

$$8x + 6xy^2 \frac{dy}{dx} + 2y^3 - 10y \frac{dy}{dx} = 0$$

$$6xy^2 \frac{dy}{dx} - 10y \frac{dy}{dx} = -8x - 2y^3$$

$$\frac{dy}{dx} (6xy^2 - 10y) = -8x - 2y^3$$

$$\therefore \frac{dy}{dx} = \frac{-8x - 2y^3}{6xy^2 - 10y}$$

When $x = 1$ and $y = 2$

$$\frac{dy}{dx} = \frac{-8 - 2y}{6(1)(2^2) - 10(2)} = \frac{-24}{4}$$

$$\therefore \frac{dy}{dx} = -6$$