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Matric No: 19/MHS01/352

1. $\int \frac{2x}{\sqrt{4x^2-1}}\,dx$

Let $u = 4x^2 - 1$

$du/dx = 8x$

$dx = \frac{du}{8x}$

$\int \frac{2x}{\sqrt{4x^2-1}}\,dx = \int \frac{2x}{u^{1/2}}\,dx \Rightarrow \int \frac{2x}{u^{1/2}} \cdot \frac{du}{8x}$

$= \frac{2x}{8x}\int \frac{du}{u^{1/2}}$

$= \frac{1}{4}\int \frac{1}{u^{1/2}}\,du$

$= \frac{1}{4}\int \frac{u^{-1/2+1}}{-1/2+1} + C$

$= \frac{1}{4}\left[\frac{u^{1/2}}{1/2}\right] + C$

$= \frac{1}{4}\left(2u^{1/2}\right) + C$

$= 1/2\, u^{1/2} + C$

$\int \frac{2x}{\sqrt{4x^2-1}}\,dx = 1/2\,(4x^2-1)^{1/2} + C$ or $1/2\sqrt{4x^2-1} + C$

2. $\int \frac{\sin^{-1}x}{\sqrt{1-x^2}}\,dx$

Let $u = \sin^{-1}x$

$du/dx = \frac{1}{\sqrt{1-x^2}} \Rightarrow dx = du\sqrt{1-x^2}$

$\int \frac{\sin^{-1}x}{\sqrt{1-x^2}}\,dx = \int \frac{u}{\sqrt{1-x^2}} \cdot du\sqrt{1-x^2}$

$$= \int u\,du$$

$$= \frac{u^2}{2} + C$$

$$\int \frac{\sin^{-1}x}{\sqrt{1-x^2}}\,dx = \frac{(\sin^{-1}x)^2}{2} + C$$

3. $\int (\tan x)^6 \sec^2 x\,dx$

let $u = \tan x$

$du/dx = \sec^2 x \Rightarrow dx = \frac{du}{\sec^2 x}$

$$\int (\tan x)^6 \sec^2 x\,dx = \int u^6 \cancel{\sec^2 x} \cdot \frac{du}{\cancel{\sec^2 x}} = \int u^6 \cdot du$$

$$= \left[\frac{u^{6+1}}{6+1}\right] + C$$

$$= \frac{u^7}{7} + C$$

$$\int (\tan x)^6 \sec^2 x\,dx = \frac{(\tan x)^7}{7} + C$$