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1) $\int \frac{2x}{\sqrt{4x^2-1}}\,dx = \frac{1}{2}\sqrt{4x^2-1}+c$

Solution

$\int \frac{2x}{\sqrt{4x^2-1}}\,dx$

Let $u = \sqrt{4x^2-1}$ $\quad P = 4x^2-1$

$u = \sqrt{P}$

$\frac{dP}{dx} = 8x$, $\frac{dy}{dP} = P^{1/2} = \frac{1}{2}P^{-1/2} = \frac{1}{2}\times\frac{1}{\sqrt{P}} = \frac{1}{2}\times\frac{1}{\sqrt{4x^2-1}}$

$\frac{dy}{dx} = \frac{dP}{dx}\times\frac{dy}{dP} = 8x\times\frac{1}{2\sqrt{4x^2-1}}$

$\frac{du}{dx} = \frac{4x}{\sqrt{4x^2-1}}$

$dx = \frac{\sqrt{4x^2-1}}{4x}\,du$

$dx = \frac{\sqrt{4x^2-1}}{4u}\,du$

$\int \frac{2x\times\sqrt{4x^2-1}}{\sqrt{4x^2-1}\times 4x}\,du$

$\int \frac{1}{2}\,du$

$\frac{1}{2}\int du = \frac{1}{2}u + c$

$\frac{1}{2}\times\sqrt{4x^2-1}+c$

2 $\int \frac{\sin^{-1}x}{\sqrt{1-x^2}}\,dx = (\sin^{-1}x\,x - \frac{(\sin^{-1}x)^2}{2}+c$

Solution

$\int \frac{\sin^{-1}x}{\sqrt{1-x^2}}\,dx$

let $u = \sqrt{1-x^2}$, $p = 1-x^2$

$u = \sqrt{p}$, $du/dp = \frac{1}{2}p^{-1/2}$ $dp/dx = -2x$

$$\frac{du}{dx} = \frac{dp}{dx} \times \frac{du}{dp}$$

$$\frac{du}{dx} = -2x \times \frac{1}{2} \times \frac{1}{\sqrt{p}}$$

$$\frac{du}{dx} = \frac{-x}{\sqrt{p}} = \frac{-x}{\sqrt{1-x^2}}$$

$$dx = -\frac{\sqrt{1-x^2}}{x}\,du$$

from the question

$$\int \frac{\sin^{-1}x}{\sqrt{1+x^2}}\,dx$$

$$\int \frac{\sin^{-1}x}{\sqrt{1-x^2}} x - \frac{\sqrt{1-x^2}}{x}\,du$$

$$= -\int \frac{\sin^{-1}x\,du}{x}$$

But $u = \sqrt{1-x}$

$u^2 = 1-x$

$x = 1-u^2$

$$= \int \frac{\sin^{-1}(1-u^2)\,du}{1-u^2} = \ln(\sin^{-1}x) + c$$

3 $\int (\tan x)^6 \sec^2 x\,dx$

$\begin{cases} u = \tan x \\ du = \sec^2 x\,dx \\ u^6\,du = u^7/7 \end{cases}$

Sol^n

$\int (\tan x)^6 \sec^2 x\,dx$

let $u = \tan x$

$du/dx = \sec^2 x$ $\therefore dx = du/\sec^2 x$

$$\int u^6 \times \sec^2 x \times \frac{du}{\sec^2 x}$$

$$\int u^6 \times \sec^2 \times \frac{du}{\sec^2 x}$$

$$\int u^6 du = \frac{u^7}{7} + c$$

$$= \frac{(\tan x)^7}{7} + c$$