

1. Find $\int \frac{2x}{\sqrt{4x^2-1}} dx$. Let $z = 4x^2 - 1$

$$\frac{dz}{dx} = 8x \quad \therefore dx = \frac{dz}{8x}$$

$$= \int \frac{2x}{\sqrt{z}} \cdot \frac{dz}{8x} = \frac{1}{4} \int \frac{1}{\sqrt{z}} dz$$

$$= \frac{1}{4} \cdot \frac{\sqrt{z}}{0.5} + C = \frac{\sqrt{4x^2-1}}{2} + C, \quad [z = 4x^2 - 1]$$

2. Find $\int \frac{\sin^{-1}(x)}{\sqrt{1-x^2}} dx$. Let $z = \sin^{-1}x$

$$\frac{dz}{dx} = \frac{1}{\sqrt{1-x^2}} \quad \therefore dx = \sqrt{1-x^2} \cdot dz$$

$$= \int \frac{z}{\sqrt{1-x^2}} \cdot \sqrt{1-x^2} \cdot dz = \int z \cdot dz$$

$$= \frac{z^2}{2} + C = \frac{(\sin^{-1}x)^2}{2} + C, \quad [z = \sin^{-1}x]$$

3. $\int (\tan x)^6 \sec^2 x dx$.

Let $z = \tan x$, $\frac{dz}{dx} = \sec^2 x$; $dz = \sec^2 x dx$

$$\therefore \int z^6 \cdot dz = \frac{z^7}{7} + C$$

$$= \frac{(\tan x)^7}{7} + C$$