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mbbs

MAT 104 assignment

1. $\int \frac{2x}{\sqrt{4x^2-1}} dx$

$$\int 2x \cdot (4x^2-1)^{-1/2} \cdot dx$$

$$\text{Let } u = 4x^2 - 1$$

$$= \frac{du}{dx} = 8x$$

$$8x dx = du \Rightarrow dx = \frac{du}{8x}$$

$$= \int 2x \cdot u^{-1/2} \cdot \frac{du}{8x}$$

$$= \frac{1}{4} \int u^{-1/2} du$$

$$= \frac{1}{4} \left[\frac{u^{1/2}}{1/2} \right] + C$$

$$= \frac{1}{4} \times 2u^{1/2} + C$$

$$= \frac{1}{2} u^{1/2} + C$$

$$= \frac{1}{2} \sqrt{4x^2-1} + C$$

$$= \frac{(\sin^{-1}x)^2}{2} + C$$

3) $\int (\tan x)^6 \sec^2 x dx$

$$u = \tan x$$

$$\frac{du}{dx} = \sec^2 x = du = \sec^2 x dx$$

$$dx = \frac{du}{\sec^2 x}$$

$$= \int u^6 \cdot \sec^2 x \cdot \frac{du}{\sec^2 x}$$

$$= \int u^6 \cdot du$$

$$= \frac{u^7}{7} + C$$

$$= \frac{(\tan x)^7}{7} + C$$

2. $\int \frac{\sin^{-1}x}{\sqrt{1-x^2}} dx$

$$\int \sin^{-1}x \cdot \frac{1}{\sqrt{1-x^2}} dx$$

$$u = \sin^{-1}x$$

$$x = \sin u$$

$$\frac{dx}{du} = \cos u$$

Recall

$$\sin^2 u + \cos^2 u = 1$$

$$\cos^2 u = 1 - \sin^2 u$$

$$\cos^2 u = 1 - x^2$$

$$\cos u = \sqrt{1-x^2}$$

$$= \frac{dx}{du} = \sqrt{1-x^2} = dx = \sqrt{1-x^2} du$$

$$= du = \frac{1}{\sqrt{1-x^2}} \cdot dx$$

$$= \int u \cdot du$$

$$= \frac{u^2}{2} + C$$