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DEPARTMENT: MECHATRONICS

MATRIC NO: 19/ENG05/001.

Assignment

1. Derivative of the following using first principle

b. $y = \frac{4}{x^3}$

Solution

$$y = \frac{4}{x^3}$$

$$y + \Delta y = \frac{4}{(x+\Delta x)^3}$$

$$\Delta y = \frac{4}{(x+\Delta x)^3} - \frac{4}{x^3}$$

$$\Delta y = \frac{4x^3 - 4(x+\Delta x)^3}{x^3(x+\Delta x)^3}$$

$$\Delta y = \frac{4x^3 - 4(x^3 + 3x^2\Delta x + 3x\Delta x^2 + \Delta x^3)}{x^3(x+\Delta x)^3}$$

$$\Delta y = \frac{\cancel{4x^3} - [\cancel{4x^3} + 12x^2\Delta x + 12x\Delta x^2 + 4\Delta x^3]}{x^3(x+\Delta x)^3}$$

$$\Delta y = \frac{-12x^2\Delta x - 12x\Delta x^2 - 4\Delta x^3}{x^3(x+\Delta x)^3}$$

$$\Delta y = \frac{-12x^2\Delta x - 12x\Delta x^2 - 4\Delta x^3}{x^3(x+\Delta x)^3}$$

$$\frac{\Delta y}{\Delta x} = \frac{-12[x^2 + x\Delta x] - 4\Delta x^2}{x^3(x+\Delta x)^3}$$

$$\frac{\Delta y}{\Delta x} = \frac{-12[x^2 + x\Delta x] - 4\Delta x^2}{x^3(x+\Delta x)^2} \times \frac{\Delta x}{\cancel{\Delta x}}$$

$$\frac{\Delta y}{\Delta x} = \frac{-12[x^2 + x\Delta x] - 4\Delta x^2}{x^3[x+\Delta x]^2}$$

$$\lim_{x \to 0} \frac{\Delta y}{\Delta x} = \frac{-12[x^2 + x[0]] - 4[0]}{x^3[x+[0]]^3}$$

$$= \frac{-12x^2}{x^3(x^3)} = \frac{-12x^2}{x^6}$$

$$= 12x^{-4}$$

1 Derivative of the following using first principle.

$y = \sin\left[\frac{3}{x^2}\right]$

Solution

$$y = \sin\left[\frac{3}{x^2}\right]$$

$$y + \Delta y = \sin \frac{3}{(x+h)^2}$$

$$\Delta y = \sin \frac{3}{(x+h)^2} - \sin \frac{3}{x^2}$$

Recall $\sin A - \sin B = 2\cos \frac{A+B}{2} \sin \frac{A-B}{2}$

$$\frac{A+B}{2} = \frac{3}{(x+h)^2} + \frac{3}{x^2}$$

$$= \frac{3x^2 + 3(x+h)^2}{2(x+h)^2 x^2}$$

$$\frac{A-B}{2} = \frac{3}{(x+h)^2} - \frac{3}{x^2}$$

$$\frac{3x^2 - 3(x+h)^2}{2(x+h)^2 x^2} = \frac{-6xh - 3h^2}{2(x+h)^2 x^2}$$

$$\frac{\Delta y}{h} = \frac{2\cos\left[\frac{3x^2 + 3(x+h)^2}{2(x+h)^2 x^2}\right] \sin\left[\frac{-6xh - 3h^2}{2(x+h)^2 x^2}\right]}{h}$$

$$2\cos\left[\frac{3x^2 + 3(x+h)^2}{2(x+h)^2 x^2}\right] \frac{\sin\left[\frac{-6xh - 3h^2}{2(x+h)^2 x^2}\right]}{h\left[\frac{-6x - 3h}{2(x+h)^2 x^2}\right]} \times \frac{-6x - 3h}{2(x+h)^2 x^2}$$

$h \to 0$

$$\cancel{2}\cos\left[\frac{3x^2 + 3(x+h)^2}{2(x+h)^2 x^2}\right] \times \frac{-6x - 3h}{\cancel{2}(x+h)^2 x^2}$$

$$\cos\left[\frac{3x^2 + 3(x+0)^2}{2(x+0)^2 x^2}\right] - \frac{6x - 3(0)}{(x+0)^2 x^2}$$

$$\cos\left[\frac{3x^2 + 3x^2}{2x^4}\right] - \frac{6x}{x^4}$$

$$\frac{-6x}{x^4} \cos \frac{6x^2}{2x^4}$$

$$= \frac{-6}{x^3} \cos \frac{3}{x^2}$$

2. Integral of the following.

i. $\frac{dx}{(x^2+36)}$

ii. $\frac{dx}{(x^2+15)}$

Solution

i. $\frac{dx}{(x^2+36)}$

$$\int \frac{dx}{x^2+36} = \int \frac{dx}{x^2+6^2}$$

$x = 6\tan\theta$

$$\frac{dx}{d\theta} = 6\sec^2\theta$$

$dx = 6\sec^2\theta \, d\theta$

$$x^2+6^2 = 6^2\tan^2\theta + 6^2$$

$$= 6^2(\tan^2\theta + 1)$$

$$= 36\sec^2\theta$$

$$= \int \frac{6\sec^2\theta \, d\theta}{36\sec^2\theta} = \int \frac{d\theta}{6} = \frac{1}{6}$$

$$= \frac{1}{6}[\theta] + c$$

$$= \frac{1}{6}\tan^{-1}\frac{x}{6} + c$$

11: $\int \frac{dx}{x^2+13}$

$$\int \frac{dx}{x^2+13} = \int \frac{dx}{(x^2+(\sqrt{13})^2}$$

$$x = \sqrt{13}\tan\theta$$

$$\frac{dx}{d\theta} = \sqrt{13}\sec^2\theta$$

$$dx = \sqrt{13}\sec^2\theta\, d\theta$$

$$[x^2+(\sqrt{13})^2] = (\sqrt{13})^2 + \tan^2\theta + (\sqrt{13})^2$$

$$= (\sqrt{13})^2[\tan^2\theta + 1]$$

$$= 13\sec^2\theta$$

$$= \int \frac{\sqrt{12}\sec^2\theta\, d\theta}{12\sec^2\theta} = \int \frac{d\theta}{\sqrt{13}} = \frac{1}{13}\int d\theta$$

$$= \frac{1}{\sqrt{12}}[0] + C$$

$$= \frac{1}{\sqrt{12}}\tan^{-1}\frac{x}{\sqrt{12}} + C$$