

$$1) \int \frac{2x}{\sqrt{4x^2-1}} dx$$

$$\text{let } u = \sqrt{4x^2-1} = (4x^2-1)^{1/2}$$

$$\frac{du}{dx} = \frac{1}{2} (4x^2-1)^{-1/2} \cdot 8x$$

$$\frac{du}{dx} = 4x(4x^2-1)^{-1/2}$$

$$dx = \frac{du}{4x(4x^2-1)^{-1/2}} = \frac{(4x^2-1)^{1/2} du}{4x}$$

we have

$$2 \int \frac{x}{u} dx = 2 \int \frac{x}{(4x^2-1)^{1/2}} \cdot \frac{(4x^2-1)^{1/2} du}{4x}$$

$$= \frac{1}{2} \int du$$

$$= \frac{1}{2} u + C = \frac{1}{2} \sqrt{4x^2-1} + C$$

$$2) \int \frac{\sin^{-1} x}{\sqrt{1-x^2}} dx$$

$$= \int \sin^{-1} x \cdot (1-x^2)^{-1/2} dx$$

$$\text{let } u = \sin^{-1} x$$

$$du = (1-x^2)^{-1/2} dx$$

$$\int u \cdot du = \frac{u^2}{2} + C$$

$$= \frac{(\sin^{-1} x)^2}{2} + C$$

$$3. \int (\tan x)^6 \sec^2 x dx$$

$$\text{let } u = \tan x$$

$$du = \sec^2 x dx$$

we have

$$\int u^6 du = \frac{u^7}{7} + C$$

$$= \frac{(\tan x)^7}{7} + C$$