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ARTMENT:- MICROBIOLOGY
ZIC NO:- 19/SCI05/004
SE CODE: MAT 104

Assignment

$y = \sin(3/x^2)$

$y + \Delta y = \sin\left[\frac{3}{(x+\Delta x)^2}\right] - y$

$\Delta y = \sin\left[\frac{3}{(x+\Delta x)^2}\right] - y$

$\Delta y = \sin\left[\frac{3}{(x+\Delta x)^2}\right] - \sin\left[\frac{3}{x^2}\right]$ --- (1)

Recall, $\sin A - \sin B = 2\cos\frac{(A+B)}{2}\sin\frac{(A-B)}{2}$ --- (2)

Comparing equation (1) & (2)

$A = \frac{3}{(x+\Delta x)^2}$ and $B = \left[\frac{3}{x^2}\right]$

$\frac{A+B}{2} = \frac{3}{(x+\Delta x)^2} + \frac{3}{x^2}$

$\Rightarrow \frac{3x^2 + 3(x+\Delta x)^2}{(x+\Delta x)^2(x^2)} = 2$

$\Rightarrow \frac{3x^2 + 3(x+\Delta x)(x+\Delta x)}{(x+\Delta x)^2(x^2)} = 2$

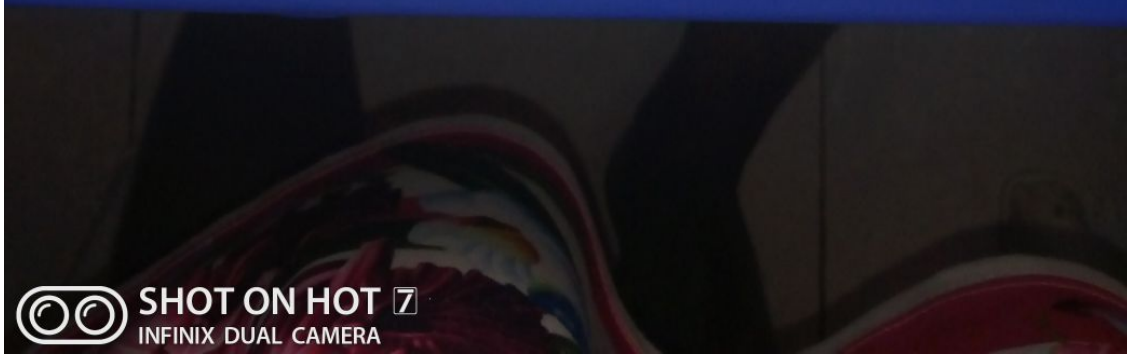
$\Rightarrow \frac{3x^2 + 3x + 3\Delta x(x+\Delta x)}{(x+\Delta x)^2(x^2)} = 2$

$\Rightarrow \frac{3x^2 + 3x^2 + 3x\Delta x + 3x\Delta x + 3(\Delta x)^2}{(x+\Delta x)^2(x^2)} = 2$

$\Rightarrow \frac{9x^2 + 9x\Delta x + 3(\Delta x)^2}{(x+\Delta x)^2(x^2)} \times \frac{1}{2}$

$\Rightarrow \frac{9x^2 + 9x\Delta x + 3(\Delta x)^2}{2[(x+\Delta x)^2(x^2)]}$

And $\frac{A-B}{2} = \frac{\left[\frac{3}{(x+\Delta x)^2}\right] - \left[\frac{3}{x^2}\right]}{2}$



2) $\int \frac{dx}{(x^2+36)}$

Let $u = (x^2+36)$

$\frac{du}{dx} = 2x \Rightarrow dx = \frac{du}{2x}$

$\int \frac{dx}{(x^2+36)} = \int \frac{1}{u}\left(\frac{du}{2x}\right) = \frac{1}{2x}\int \frac{du}{u}$

$= \frac{1}{2x}\left[\frac{u^2}{2}\right] + 2$

$= \frac{1}{4x}\left[u^2\right] + 2$

$= \frac{1}{4x}(x^2+36)^2 + c$

3) $\int \frac{dx}{(x^2+13)}$

let $u = (x^2+13)$

$\frac{du}{dx} = 2x$

$dx = \frac{du}{dx}$

$\int \frac{dx}{x^2+13} = \int \frac{1}{u}\left(\frac{du}{2x}\right) \Rightarrow \frac{1}{2x}\int \frac{du}{u}$

$= \frac{1}{2x}\left[\frac{u^2}{2}\right] + 2$

$= \frac{1}{4x}u^2 + 2$

$= \frac{1}{4x}(x^2+13)^2 + c$

(2)

$$\frac{A-B}{2} = \frac{\frac{3}{(x+\Delta x)^2} - \frac{3}{x^2}}{2}$$

$$\frac{3x^2 - 3(x+\Delta x)^2}{(x+\Delta x)^2 (x^2)} \div 2$$

$$= \frac{3x^2 - 3(x+\Delta x)(x+\Delta x)}{(x+\Delta x)^2 (x^2)}$$

$$= \frac{3x^2 - (3x^2 + 3\Delta x)(x+\Delta x)}{(x+\Delta x)^2 (x^2)}$$

$$= \frac{3x^2 - (3x^2 + 3x\Delta x + 3x\Delta x + \dots}{(x+\Delta x)^2 (x^2)}$$

$$= \frac{3x^2 - (3x^2 + 9x\Delta x + 3\Delta x)^2}{(x+\Delta x)^2 (x^2)}$$

$$= \frac{3x^2 - 3x^2 - 9x\Delta x - 3(\Delta x)^2}{(x+\Delta x)^2 (x^2)}$$

$$= \frac{-9x\Delta x - 3(\Delta x)^2}{2[(x+\Delta x)^2 \cdot (x^2)]}$$

Hence $\sin\left[\frac{3}{(x+\Delta x)^2}\right] - \sin\left[\frac{3}{x^2}\right]$

$$2\cos\left[\frac{9x^2 + 9x\Delta x + 3(\Delta x)^2}{2[(x+\Delta x)^2 (x^2)]}\right] \sin\left[\frac{-9x\Delta x - 3(\Delta x)^2}{2(x+\Delta x)^2 (x^2)}\right] \times \frac{1}{2}$$

$$\left[\frac{9x\Delta x - 3(\Delta x)^2}{2((x+\Delta x)^2 (x^2)}\right]$$

Divide through by Δx

$$2\cos\left[\frac{9x^2 + 9x\Delta x + 3(\Delta x)^2}{2[(x+\Delta x)^2 (x^2)}\right] \sin\left[\frac{-9x\Delta x - 3(\Delta x)^2}{2(x+\Delta x)^2 (x^2)}\right] \times \frac{1}{2}$$

$\lim x \to 0$

$$\cos\left[\frac{9x^2 + 0 + 0}{2x^4}\right] \sin\left[\frac{0}{2x^4}\right] / x/2$$

Hence, $\frac{dy}{dx} = \cos\frac{9x^2}{2x^4}$

$y = \frac{4}{x^3}$

$y + \Delta y = \frac{4}{(x+\Delta x)^3}$

$\Delta y = \frac{4}{x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + \Delta x^3} - y$

$= \frac{4}{x^3 + 3x^3\Delta x + 3x(\Delta x)^2 + \Delta x^3} - \frac{4}{x^2}$

$= \frac{4x^3 - 4(x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + \Delta x^3)}{(x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + \Delta x^3)x^3}$

$= \frac{4x^3 - (x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + \Delta x^3)}{(x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + \Delta x^3)x^3}$

$= \frac{4x^3 - x^3 - 3x^2\Delta x - 3x(\Delta x)^2 - \Delta x^3}{(x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + \Delta x^3)x^3}$

Divide both sides by Δx

$= \frac{4(-3x^2\Delta x - 3x(\Delta x)^2 - \Delta x^3) \div \Delta x}{(x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + \Delta x^3)x^3}$

$= \frac{4(-3x^2\Delta x - 3x(\Delta x)^2 - \Delta x^3) \times \frac{1}{\Delta x}}{(x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + \Delta x^2)x^3}$

$= \frac{4(-3x^2 - 3x\Delta x - \Delta x^2)\Delta x \times \frac{1}{\Delta x}}{(x^2 + 3x^2(\Delta x) + 3x(\Delta x)^2 + \Delta x^2)x^3}$

$\frac{\Delta y}{\Delta x} = \frac{4(-3x^2 - 3x\Delta x - \Delta x^2)}{x^2 + 3x^2\Delta x + 3(\Delta x + \Delta x^3)}$

$\lim \Delta x \to 0$

$\frac{dy}{dx} = \frac{4(-3x^2)}{(x^2+0)x^2}$

$\frac{dy}{dx} = \frac{-12x^2}{x^4}$

$= \frac{-12}{x^2}$