

1a) Charging by induction: Electric charge can be obtained on an object without touching it by a process called induction. In such a process, a positively charged rubber rod is brought near a neutral conducting sphere which is insulated. The repulsive force between the positive charges in the sphere and rod causes redistribution of charges in the sphere. The positive charge moves to the farthest end of the sphere while the negative charge remains close to the positively charged rod. By grounding the sphere, the excess positive charge is removed. Finally when the rubber is removed, the negative electrons becomes uniformly distributed.

See figure



$$1b. k = 9 \times 10^9$$

$$Q_1 + Q_2 = 5 \times 10^{-5} C$$

$$F = 1 N \quad d = 2 m$$

$$F = \frac{k Q_1 Q_2}{r^2}$$

$$1 = \frac{9 \times 10^9 \times (Q_1 Q_2) \times 10^{-5}}{2^2}$$

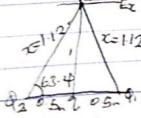
$$1 = 9 \times 10^9 \times 5 \times 10^{-5} Q_1 + 9 \times 10^9 Q_2$$

$$9 \times 10^9 Q_2 - 4.5 \times 10^5 Q_1 + 4 = 0$$

$$Q_1 = 0.0000111 C = 1.11 \times 10^{-5} C$$

$$Q_2 = 0.0000382 C = 3.8 \times 10^{-5} C$$

$$1c. Q_1 = Q_2 = 8 \mu C \quad d = 0.5 m$$



$$\tan \theta = \frac{x}{z}$$

$$\theta = \tan^{-1}(0.5)$$

$$= 63.4^\circ$$

$$z^2 = x^2 + 0.5^2$$

$$x = \sqrt{z^2 - 0.5^2} = 1.12$$

$$E_x = \frac{k Q_2}{r^2} = \frac{9 \times 10^9 \times 8 \times 10^{-6}}{1.12^2} = 5739.795918 V$$

$$E_y = \frac{k Q_1}{r^2} = \frac{9 \times 10^9 \times 8 \times 10^{-6}}{1.12^2} = 5739.795918 V$$

$$E_{\text{tot}} = k Q_1 = \frac{k Q_2}{r^2} = \frac{9 \times 10^9 \times 8 \times 10^{-6}}{1} = 9 \times 10^9 V$$

| Vector | Angle | x-comp | y-comp |
|------------------------------------|-------|----------------------|----------------------|
| $E_x = 5739.795918$ | 63.4 | $E_x \cos 63.4$ | $E_x \sin 63.4$ |
| $E_y = 5739.795918$ | 63.4 | $E_y \cos 63.4$ | $E_y \sin 63.4$ |
| $E_{\text{tot}} = 9 \times 10^9 V$ | 90° | $E_{\text{tot}} = 0$ | $E_{\text{tot}} = 0$ |

$$\text{Magnitude} = \sqrt{E_x^2 + E_y^2}$$

$$= \sqrt{0^2 + (9 \times 10^9)^2} = 9 \times 10^9$$

$$\text{Since } \theta = 0$$

$$0 = 9 \times 10^9 Q_1 + 1.12 \times 10^9 \cdot 5739.795918$$

$$Q_1 = -1.0264 \cdot 5739.795918$$

$$Q_1 = -5.8 \times 10^9 C$$

$$Q_1 = 1.1405028 C$$

$$Q_1 = 1.1405028 C$$

3b) Electric Potential difference: The electric potential difference between two points is said to be the work done per unit charge against electrical forces when a charge is transported from one point to the other. It is measured in volt (V) or joules per coulomb (J/C). It is a scalar quantity.

Considering this diagram, a test charge q_0 is moved from point A to point B along an arbitrary path inside an electric field E . The electric field E exerts a force $F = q_0 E$ on the charge q_0 shown in the diagram. To move the test charge from A to B at constant velocity, an external force of $F = -q_0 E$ must act on the charge. Therefore, the external work done dW is given as $dW = F \cdot dl = -q_0 E \cdot dl$.

$$\text{But } F = -q_0 E \quad \text{--- (2)}$$

Substituting equation 2 in 1

$$dW = -q_0 E dl \quad \text{--- (3)}$$

Then total work done in moving the test charge from A to B is

$$W(A \rightarrow B) = -q_0 \int_A^B E dl \quad \text{--- (4)}$$

From the definition of epd, it follows that

$$V_B - V_A = W(A \rightarrow B) \quad \text{--- (5)}$$

Putting 4 in 5

$$V_B - V_A = - \int_A^B E dl \quad \text{--- (6)}$$

3a) (i) Volume charge density $\rho = \frac{dQ}{dv} \rightarrow dQ = \rho dv$

(ii) Surface charge density $\sigma = \frac{dQ}{dA} \rightarrow dQ = \sigma dA$

(iii) Linear charge density $\lambda = \frac{dQ}{dl} \rightarrow dQ = \lambda dl$

4a. Magnetic flux is defined as the strength of the magnetic field which can be represented by lines of forces. It is represented by the symbol Φ . Mathematically $\Phi = B \cdot dA$

$$4b. m = 9 \times 10^{-31} \text{ kg} \quad r = 1.4 \times 10^{-3} \text{ m}$$

$$B = 3.5 \times 10^{-1} \text{ weber/m}$$

From $v = rw$

$$w = v/r = \frac{qB}{m} = 1.6 \times 10^{-19} \times 3.5 \times 10^{-1}$$

$$w = 5.6 \times 10^{19} \text{ T}^{-1}$$

4c. In the question, parameters such as (i) mass of electron = $9.11 \times 10^{-31} \text{ kg}$
 (ii) radius of $1.4 \times 10^{-3} \text{ m}$
 (iii) magnetic field of $3.5 \times 10^{-1} \text{ weber/meter square}$
 and we were asked to find the cyclotron frequency which is ω_c or angular speed. It is so called because it is a frequency of an accelerator called cyclotron.

Recall that angular speed is given as $\omega = v/r = qB/m$
 Substituting we have $\omega = v/r = \frac{qB}{m} = 1.6 \times 10^{-19} \times 3.5 \times 10^{-1}$

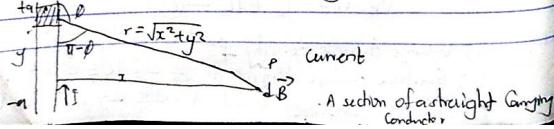
Then we get our answer - have T^{-1} as it was since angular speed is same as cyclotron frequency.

5a. Biot-Savart law states that the magnetic field is directly proportional to the product permeability of free space (μ_0) the current (I), change in length, the radius and inversely proportional to square of radius (r^2). Mathematically $d\vec{B} = \frac{\mu_0 I}{4\pi r^2} dI \hat{r}$

where μ_0 is a constant called Permeability of free space.
 $\mu_0 = 4\pi \times 10^{-7} \text{ T m/A}$

The unit of B is weber/meter square

5b. Magnetic field of a straight current carrying conductor



The magnetic field of B Applying Biot-Savart law we find the magnitude of the field B

$$B = \frac{\mu_0 I}{4\pi r^2} \int_{-a}^a \frac{dI \sin(\theta)}{r^2}$$

$$\sin(\theta) = \sin \theta$$

$$\therefore B = \frac{\mu_0 I}{4\pi r^2} \int_{-a}^a \frac{dI \sin(\theta)}{r^2}$$

From the diagram $r^2 = x^2 + y^2$ (Pythagoras theorem)

$$B = \frac{\mu_0 I}{4\pi r^2} \int_{-a}^a \frac{dI \sin(\theta)}{x^2 + y^2}$$

$$\text{But } \sin(\theta) = \frac{x}{\sqrt{x^2 + y^2}} = \frac{x}{(x^2 + y^2)^{1/2}}$$

Substituting (**) into (**)

$$B = \frac{\mu_0 I}{4\pi r^2} \int_{-a}^a \frac{dI}{x^2} \frac{x}{(x^2 + y^2)^{1/2}}$$

$$B = \frac{\mu_0 I}{4\pi r^2} \int_{-a}^a \frac{dI}{(x^2 + y^2)^{3/2}} x$$

$$\text{Recall } dI = dy \quad \therefore B = \frac{\mu_0 I}{4\pi r^2} \int_{-a}^a \frac{x}{(x^2 + y^2)^{3/2}} dy$$

$$B = \frac{\mu_0 I x}{4\pi r^2} \int_{-a}^a \frac{1}{(x^2 + y^2)^{3/2}} dy \quad ***$$

Using Special integrals.

$$\int \frac{dy}{(x^2 + y^2)^{3/2}} = \frac{1}{x} \frac{y}{(x^2 + y^2)^{1/2}}$$

Equation (***) therefore becomes

$$B = \frac{\mu_0 I x}{4\pi r^2} \left[\frac{y}{x^2 + y^2} \right]_{-a}^a$$

$$B = \frac{\mu_0 I x}{4\pi r^2} \left[\frac{2a}{x^2 + a^2} \right]$$

$$B = \frac{\mu_0 I}{4\pi r^2} \left(\frac{2a}{x^2 + a^2} \right)$$

When the length $2a$ of the conductor is very great in comparison to its distance x from point P , we consider it infinitely long. That is when a is much larger than x , $(x^2 + a^2)^{1/2} \approx a$, or $a \rightarrow \infty \rightarrow 0 \quad \therefore B = \frac{\mu_0 I}{2\pi r^2}$

In a physical situation, we have axial symmetry about the y -axis. Thus, at all points in a circle of radius r around the conductor, the magnitude of B is $B = \frac{\mu_0 I}{2\pi r}$ (#)

Equation # defines the magnitude of the magnetic field of flux density B near a long straight current carrying conductor.