INYANG MARTIN VICTOR 19/SCI01/053

1 Computer Science.

1a. $y = \sin\left(3/x^2\right)$ from first principle.

$y + \Delta y = \sin\left[\frac{3}{(x+\Delta x)^2}\right]$

$\Delta y = \sin\left[\frac{3}{(x+\Delta x)^2}\right] - y$

$\Delta y = \sin\left[\frac{3}{(x+\Delta x)^2}\right] - \left[\sin\left(\frac{3}{x^2}\right)\right]$ — (1)

Recall $\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$ — (2)

Comparing equation 1 & 2

$A = \frac{3}{(x+\Delta x)^2}$ and $B = \left[\frac{3}{x^2}\right]$

$\frac{A+B}{2} = \frac{3}{(x+\Delta x)^2} + \frac{3}{x^2} = \frac{3x^2 + 3(x+\Delta x)^2}{(x+\Delta x)^2(x^2)} \div 2$

$\frac{3x^2 + 3(x+\Delta x)(x+\Delta x)}{(x+\Delta x)^2(x^2)} \div 2$

$= \frac{3x^2 + 3x + 3\Delta x(x+\Delta x)}{(x+\Delta x)^2(x^2)} \div 2$

$= \frac{3x^2 + 3x^2 + 3x\Delta x + 3x\Delta x + 3(\Delta x)^2}{(x+\Delta x)^2(x^2)} \div 2$

$= \frac{9x^2 + 9x\Delta x + 3(\Delta x)^2}{(x+\Delta x)^2(x^2)} \times \frac{1}{2}$

$= \frac{9x^2 + 9x\Delta x + 3(\Delta x)^2}{2[(x+\Delta x)^2(x^2)]}$.

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b $y = \frac{4}{x^3}$

$y + \Delta y = \frac{4}{(x+\Delta x)^3}$

$y + \Delta y = \frac{4}{x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + \Delta x^3}$

$\Delta y = \frac{4}{x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + \Delta x^3} - y$

$= \frac{4}{x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + \Delta x^3} - \frac{4}{x^3}$

$= \frac{4x^3 - 4(x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + \Delta x^2)}{(x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + \Delta x^3)x^3}$

$= \frac{4x^3 - (x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + \Delta x^3)}{(x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + \Delta x^3)x^3}$

$\Delta y = \frac{4x^3 - x^3 - 3x^2\Delta x - 3x(\Delta x)^2 - \Delta x^3}{(x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + \Delta x^3)x^3}$

Divide both sides by Δx

$\frac{\Delta y}{\Delta x} = \frac{4(-3x^2\Delta x - 3x(\Delta x)^2 - \Delta x^3)}{(x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + \Delta x^3)x^3} \div \frac{\Delta x}{1}$

$= \frac{4(-3x^2\Delta x - 3x(\Delta x)^2 - \Delta x^3)}{(x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3)x^3} \times \frac{1}{\Delta x}$

$= \frac{4(-3x^2 - 3x\Delta x - \Delta x^2)\Delta x}{(x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3)x^3} \times \frac{1}{\Delta x}$

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$$\frac{\Delta y}{\Delta x} = \frac{4(-3x^2 - 3x\Delta x - \Delta x^2)}{(x^2 + 3x^2\Delta x + 3x(\Delta x)^2 + \Delta x^3)\,x^2}$$

as $\Delta x \to 0$

$$\frac{\Delta y}{\Delta x} = \frac{4(-3x^2)}{(x^2)(x^2)}$$

$$\frac{\Delta y}{\Delta x} = \frac{-12x^2}{x^4} = \frac{-12}{x^2}$$

2a Find the integral of the ff.

a $\int \frac{dx}{(x^2 + 36)}$

Let $u = (x^2 + 36)$

$\frac{du}{dx} = 2x$

$dx = \frac{du}{2x}$

$$\int \frac{dx}{x^2+36} = \int \frac{1}{u}\frac{du}{2x}$$

$$= \frac{1}{2x}\int \frac{du}{u} = \frac{1}{2x}\left[\frac{u^2}{2}\right] + 2$$

$$= \frac{1}{4x}\left[u^2\right] + 2$$

$$= \frac{1}{4x}(x^2 + 36)^2 + c$$

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2 $\int \frac{dx}{(x^2+13)}$

let $u = (x^2+13)$

$\frac{du}{dx} = 2x$

$dx = \frac{du}{2x}$

$\int \frac{dx}{x^2+13} = \int \frac{1}{u}\left(\frac{du}{2x}\right)$

$= \frac{1}{2x}\int \frac{du}{u}$

$= \frac{1}{2x}\left[\frac{u^2}{2}\right] + 2$

$= \frac{1}{4x} u^2 + 2$

$= \frac{1}{4x}(x^2+13)^2 + c$

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