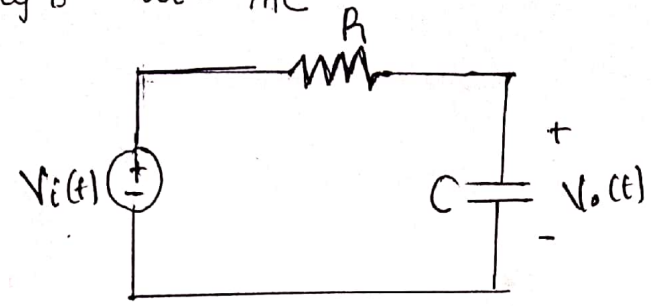
OBARAKPO EMMANUEL .C.
17/ENG05/024
MECHATRONICS ENGINEERING.

EEE322 Ass.

**Quest:** Determine the type of filter shown below, and show that its cut-off frequency is $\omega_c = 1/RC$




**Sol:** The above filter is a Low pass filter and the output is taken of the capacitor.

Applying voltage division to the frequency-domain equivalent circuit;

$$V_o = \left(\frac{1/j\omega C}{R + 1/j\omega C}\right) \times V_i$$

$$\therefore H(\omega) = \frac{V_o}{V_i} = \frac{1/j\omega C}{R + 1/j\omega C} = \frac{1/j\omega C}{\frac{j\omega RC + 1}{j\omega C}} = \boxed{\frac{1}{j\omega RC + 1}}$$

$$\therefore H(\omega) = \frac{1}{j\omega RC + 1}$$

The cutoff-frequency $\omega_c$ is obtained by setting the magnitude of $H(\omega)$ to $\frac{1}{\sqrt{2}}$

$$\therefore |H(\omega)| = \left|\frac{1}{j\omega RC + 1}\right| = \frac{1}{\sqrt{2}}$$

$$= \frac{\sqrt{1^2}}{\sqrt{(\omega RC)^2 + 1^2}} = \frac{1}{\sqrt{2}}$$

$$= \frac{1}{\sqrt{(\omega RC)^2 + 1}} = \frac{1}{\sqrt{2}}$$

$$\therefore \sqrt{(\omega RC)^2 + 1} = \sqrt{2}$$

Equating both sides; $2 = 1 + \omega^2 R^2 C^2$

$$\omega^2 = \frac{1}{R^2 C^2}$$

$$\boxed{\omega_c = \frac{1}{RC}}$$