AZEEZ ASHRAF OLASUBOMI
MECHATRONICS ENGINEERING
19/ENG05/017

DR OYEGAMI
MAT 104
April 16th, 2020

1. Find the derivation of the following using first principle

a. $y = \sin 3/x^2$

solution

$y = \sin 3/x^2$

$y + \Delta y = \sin \frac{3}{(x+\Delta x)^2}$

$\Delta y = \sin \frac{3}{(x+\Delta x)^2} - \sin \frac{3}{x^2}$

Recall $\sin A - \sin B = 2\cos \frac{A+B}{2} \sin \frac{A-B}{2}$

$A = \frac{3}{(x+\Delta x)^2}$ $\quad B = \frac{3}{x^2}$

$\frac{A+B}{2} = \frac{3}{(x+\Delta x)^2} + \frac{3}{x^2}$

$\frac{3x^2 + 3(x+\Delta x)^2}{2(x+\Delta x)^2 x^2}$

$\frac{A-B}{2} = \frac{3}{(x+\Delta x)^2} - \frac{3}{x^2}$

$\frac{3x^2 - 3(x+\Delta x)^2}{2(x+\Delta x)^2 x^2} = \frac{-6x\Delta x - 3\Delta x^2}{2(x+\Delta x)^2 x^2}$

$$\frac{\Delta y}{\Delta x} = \frac{2\cos\left[\frac{3x^2+3(x+\Delta x)^2}{2(x+\Delta x)^2 x^2}\right] \sin\left[\frac{-6x\Delta x - 3\Delta x^2}{2(x+\Delta x)^2 x^2}\right]}{\Delta x}$$

$$\frac{\Delta y}{\Delta x} = 2\cos\left[\frac{3x^2+3(x+\Delta x)^2}{2(x+\Delta x)^2 x^2}\right] \frac{\sin\left[\frac{-6x\Delta x - 3\Delta x^2}{2(x+\Delta x)^2 x^2}\right]}{\Delta x\left[\frac{-6x-3\Delta x}{2(x+\Delta x)^2 x^2}\right]} \times \frac{-6x - 3\Delta x}{2(x+\Delta x)^2 x^2}$$

$\Delta x \longrightarrow 0$

$$\cancel{2}\cos\left[\frac{3x^2+3(x+0)^2}{2(x+0)^2 x^2}\right] \times \frac{-6x - 3(0)}{\cancel{2}(x+0)^2 x^2}$$

$$\frac{dy}{dx} = \cos\left[\frac{6x^2}{2x^4}\right] \times \frac{-6x}{x^4}$$

$$\frac{dy}{dx} = \frac{-6}{x^3} \cos \frac{3}{x^2}$$

$$\frac{dy}{dx} = -6x^{-3} \cos 3/x^2$$

2 $y = 4/x^3$

$$y + \Delta y = \frac{4}{(x+\Delta x)^3}$$

$$\Delta y = \frac{4}{(x+\Delta x)^3} - \frac{4}{x^3}$$

$$\Delta y = \frac{4x^3 - 4\left[x^3 + 3x^2\Delta x + 3x\Delta x^2 + \Delta x^3\right]}{(x+\Delta x)^3\, x^3}$$

$$\frac{\Delta y}{\Delta x} = \frac{-12x^2\Delta x - 12x\Delta x^2 - 4\Delta x^3}{\Delta x\,(x+\Delta x)^3\, x^3}$$

$$\frac{\Delta y}{\Delta x} = \frac{-12x^2 - 12x\Delta x - 4\Delta x^2}{(x+\Delta x)^3\, x^3}$$

$\Delta x \longrightarrow 0$

$$\frac{dy}{dx} = \frac{-12x^2 - 12x(0) - 4(0)^2}{(x+0)^3\, x^3}$$

$$\frac{dy}{dx} = \frac{-12x^2}{x^6}$$

$$\frac{dy}{dx} = \frac{-12}{x^4}$$

$$\frac{dy}{dx} = -12x^{-4}$$

Find the integral of the following

2\. a) $\int \frac{dx}{x^2+36}$

Solution

$$\int \frac{dx}{x^2+36}$$

$x = 6\tan\theta$

$\frac{dx}{d\theta} = 6\sec^2\theta$

$dx = 6\sec^2\theta \, d\theta$

$x^2+36 = 6^2\tan^2\theta + 6^2$

$= 6^2\sec^2\theta$

$$\int \frac{6\sec^2\theta \, d\theta}{6^2\sec^2\theta}$$

$\frac{1}{6}[\theta]$

$\therefore \int \frac{dx}{x^2+36} = \frac{1}{6}\tan^{-1}\frac{x}{6} + C$

b. $\int \frac{dx}{x^2+13}$

$x = \sqrt{13}\tan\theta$

$\frac{dx}{d\theta} = \sqrt{13}\sec^2\theta$

$dx = \sqrt{13}\sec^2\theta \, d\theta$

$x^2+13 = 13\tan^2\theta + 13$

$x^2+13 = 13\sec^2\theta$

$$\int \frac{\sqrt{13}\sec^2\theta \, d\theta}{13\sec^2\theta}$$

$\frac{\sqrt{13}}{13}[\theta]$

$\frac{\sqrt{13}}{13}\tan^{-1}\frac{x}{\sqrt{13}} + C$