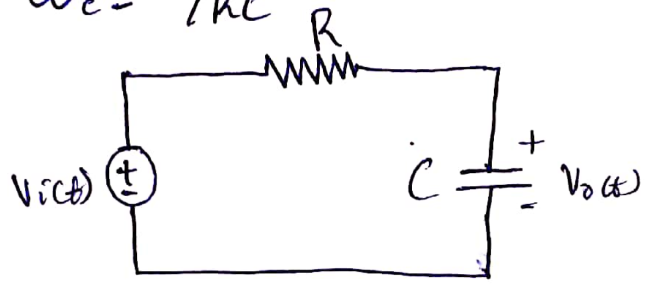
NAME: ORILADE EMMANUEL, O.
DEPT: MECHATRONICS ENGINEERING
MATRIC NO: 17/ENG05/037
ASSIGNMENT: Electric Circuit Theory II

1. Determine the type of filter shown below, and show that its cut-off frequency is $\omega_c = 1/RC$

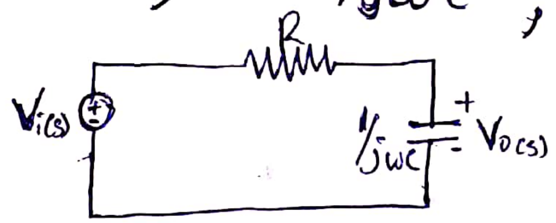


Solution

Convert the circuit from the "time domain to the frequency domain".

$V_i(t) \Rightarrow V_i(s)$; $R \Rightarrow R$; $C = 1/j\omega C$; $V_o(t) \Rightarrow V_o(s)$

$\therefore$



Using voltage division rule we would find the voltage across the capacitor; $V_o(s)$

$$\therefore V_o(s) = \left(\frac{1/j\omega C}{1/j\omega C + R}\right) \times V_i(s) \Rightarrow \left(\frac{1}{\cancel{j\omega C}} \times \frac{\cancel{j\omega C}}{1 + Rj\omega C}\right) \times V_i(s)$$

$$\therefore V_o(s) = \left(\frac{1}{1 + Rj\omega C}\right) \times V_i(s)$$

The transfer function of the filter will be;

$$H(\omega) = \frac{V_o}{V_i} = \frac{1}{1 + Rj\omega C}$$

@ $H(0) = 1/1 = 1$

@ $H(\infty) = 1/\infty = 0$

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The cut-off frequency ω_c is obtained by equating the magnitude of $H(\omega)$ to $1/\sqrt{2}$

$$\therefore |H(\omega)| = \left| \frac{1}{1+j\omega RC} \right| \Rightarrow \frac{\sqrt{1^2}}{\sqrt{1^2+|\omega^2R^2C^2|}} = \frac{1}{\sqrt{1+|\omega^2R^2C^2|}}$$

$$|H(\omega)| = 1/\sqrt{2} \quad \therefore \frac{1}{\sqrt{1+|\omega^2R^2C^2|}} = \frac{1}{\sqrt{2}}$$

$$\therefore \sqrt{2} = \sqrt{1+|\omega^2R^2C|}$$

By squaring both sides, we have;

$$2 = 1+|\omega^2R^2C^2|$$

$$1 = \omega^2R^2C^2$$

$$\therefore \omega^2 = \frac{1}{R^2C^2}$$

$$\therefore \omega = \frac{1}{RC}$$

$\therefore$ The cut-off frequency, $\omega_c = 1/RC$

ii) **NOTE:** The filter is a low-pass filter. This is because the output of ~~the~~ RC series circuit is taken off the capacitor as show in the circuit diagram