OKOH ELIJAH EROMOSELE AYOMIKUN
19/ENG05/048
MECHATRONICS ENGINEERING
MAT 104 ASSIGNMENT

(1) Find the derivative of the following using first principle

(a) $y = \sin(3/x^2)$

$\therefore y + \Delta y = \sin\left[\frac{3}{(x+\Delta x)^2}\right]$

$\therefore \Delta y = \sin\left[\frac{3}{(x+\Delta x)^2}\right] - y$

$\therefore \Delta y = \sin\left[\frac{3}{(x+\Delta x)^2}\right] - \sin\left(\frac{3}{x^2}\right)$

$\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$

$\therefore \Delta y = 2\cos \frac{\frac{3}{(x+\Delta x)^2} + \frac{3}{x^2}}{2} \sin \frac{\frac{3}{(x+\Delta x)^2} - \frac{3}{x^2}}{2}$

$\therefore \frac{\Delta y}{\Delta x} = \frac{2\cos \frac{3x^2 + 3(x+\Delta x)^2}{2x^2(x+\Delta x)^2} \sin \frac{3x^2 - 3(x+\Delta x)^2}{2x^2(x+\Delta x)^2}}{\Delta x}$

$\therefore \frac{\Delta y}{\Delta x} = \left[2\cos \frac{3x^2 + 3(x+\Delta x)^2}{2x^2(x+\Delta x)^2} \cdot \sin \frac{-6x\Delta x - 3(\Delta x)^2}{2x^2(x+\Delta x)^2}\right] \times \frac{1}{\Delta x}$

$\therefore \frac{\Delta y}{\Delta x} = \frac{2\cos \frac{3x^2+3(x+\Delta x)^2}{2x^2(x+\Delta x)^2} \sin \frac{-6x\Delta x - 3(\Delta x)^2}{2x^2(x+\Delta x)^2} \times \frac{-6x - 3\Delta x}{2x^2(x+\Delta x)^2}}{\Delta x\left[\frac{-6x - 3\Delta x}{2x^2(x+\Delta x)^2}\right]}$

$\therefore \frac{\Delta y}{\Delta x} = 2\cos \frac{3x^2+3(x+\Delta x)^2}{2x^2(x+\Delta x)^2} \frac{\sin \frac{-6x\Delta x - 3\Delta x^2}{2x^2(x+\Delta x)^2}}{\frac{-6x\Delta x - 3\Delta x^2}{2x^2(x+\Delta x)^2}} \times \frac{-6x - 3\Delta x}{2x^2(x+\Delta x)^2}$

$\therefore \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \cancel{2}\cos \frac{3x^2 + 3(x+0)^2}{2x^2(x+0)^2} \times \frac{-6x - 3(0)}{\cancel{2}x^2(x+0)^2}$

$\therefore \frac{dy}{dx} = -\frac{6\cancel{x}}{x^{\cancel{4}3}} \cos \frac{\cancel{6}^{3}x^{\cancel{2}}}{\cancel{2}x^{\cancel{4}}}$

$$\therefore \frac{dy}{dx} = \frac{-6}{x^3} \cos \frac{3}{x^2}$$ 4

(1b) $$y = \frac{4}{x^3}$$

$$y + \Delta y = \frac{4}{(x+\Delta x)^3}$$

$$\Delta y = \frac{4}{(x+\Delta x)^3} - y$$

$$\therefore \Delta y = \frac{4}{(x+\Delta x)^3} - \frac{4}{x^3}$$

$$\Delta y = \frac{4x^3 - 4(x+\Delta x)^3}{x^3(x+\Delta x)^3}$$

$$\therefore \Delta y = \frac{4x^3 - 4[(x+\Delta x)(x^2+2x\Delta x+\Delta x^2)]}{x^3(x+\Delta x)^3}$$

$$\therefore \Delta y = \frac{4x^3 - 4[x^3+2x^2\Delta x + x\Delta x^2 + x^2\Delta x + 2x\Delta x^2 + \Delta x^3]}{x^3(x+\Delta x)^3}$$

$$\therefore \Delta y = \frac{\cancel{4x^3 - 4x^3} - 8x^2\Delta x - 4x\Delta x^2 - 4x^2\Delta x - 8x\Delta x^2 - 4\Delta x^3}{x^3(x+\Delta x)^3}$$

$$\therefore \frac{\Delta y}{\Delta x} = \frac{-8x^2\Delta x - 4x\Delta x^2 - 4x^2\Delta x - 8x\Delta x^2 - 4\Delta x^3}{x^3(x+\Delta x)^3} \times \frac{1}{\Delta x}$$

$$\frac{\Delta y}{\Delta x} = \frac{\cancel{\Delta x}(-8x^2 - 4x\Delta x - 4x^2 - 8x\Delta x - 4\Delta x^2)}{x^3(x+\Delta x)^3} \times \frac{1}{\cancel{\Delta x}}$$

$$\therefore \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{-8x^2 - 4x(0) - 4x^2 - 8x(0) - 4(0)^2}{x^3(x+0)^3}$$

$$\frac{\Delta y}{\Delta x} = \frac{-8x^2 - 4x^2}{x^3(x^3)}$$

$$\frac{\Delta y}{\Delta x} = \frac{-8x^2 - 4x^2}{x^6}$$

$$\therefore \frac{\Delta y}{\Delta x} = \frac{-12\cancel{x^2}}{\cancel{x^6}\,4} = \frac{-12}{x^4}$$ 4 $$\therefore \frac{dy}{dx} = \frac{-12}{x^4}$$ 4

(2a) $\int \frac{dx}{x^2+36}$

let $x = a\tan\theta$

where $a = 6$

$\therefore x = 6\tan\theta$

$\frac{dx}{d\theta} = 6\sec^2\theta$

$\therefore dx = 6\sec^2\theta \, d\theta$

$\therefore x^2+36 = (6\tan\theta)^2 + 36$

$= 6^2\tan^2\theta + 36$

$= 36\tan^2\theta + 36$

$= 36(\tan^2\theta + 1)$

$36 + x^2 = 36(1+\tan^2\theta)$

Recall that $1+\tan^2\theta = \sec^2\theta$

$\therefore 36 + x^2 = 36\sec^2\theta$

$\therefore \int \frac{dx}{x^2+36} = \int \frac{\cancel{6}^{1}\cancel{\sec^2\theta}\,d\theta}{\cancel{36}_{6}\cancel{\sec^2\theta}}$

$= \frac{1}{6}\int d\theta$

$= \frac{1}{6}[\theta] + c$

$\therefore \int \frac{dx}{x^2+36} = \frac{1}{6}\tan^{-1}\frac{x}{6} + c$

(2b) $\int \frac{1}{x^2+13}\,dx = \int \frac{1}{x^2+(\sqrt{13})^2}\,dx$

let $x = a\tan\theta$

where $a = \sqrt{13}$

$\therefore x = \sqrt{13}\tan\theta$

$\therefore \frac{dx}{d\theta} = \sqrt{13}\sec^2\theta$

$\therefore dx = \sqrt{13}\sec^2\theta\, d\theta$

$\therefore 13 + x^2 = 13 + (\sqrt{13}\tan\theta)^2$

$= 13 + 13\tan^2\theta$

$= 13(1+\tan^2\theta)$

Recall that $\sec^2\theta = 1+\tan^2\theta$

$\therefore 13 + x^2 = 13\sec^2\theta$

$\therefore \int \frac{1}{x^2+13}\,dx = \int \frac{\sqrt{13}\,\cancel{\sec^2\theta}\,d\theta}{13\,\cancel{\sec^2\theta}}$

$= \frac{\sqrt{13}}{13}\int d\theta$

$= \frac{\sqrt{13}}{13}[\theta] + C$

$\therefore \int \frac{1}{x^2+13}\,dx = \frac{\sqrt{13}}{13}\tan^{-1}\frac{x}{\sqrt{13}} + C$

4