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17/ENG04/051
ELECT/ELECT

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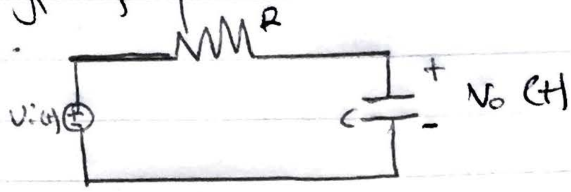
CIRCUIT THEORY ASSIGNMENT

Determine the type of filter below and show that its cut-off frequency is $W_c = \frac{1}{RC}$.



Answers

1) The diagram above is a low-pass filter as shown by the output coming out of the capacitor

2) Converting from the time domain to the frequency domain $R = R$ & $C = \frac{1}{SC}$

$H(\omega) = \frac{V_o}{V_i}$ but $V_o = \frac{1}{SC}$ and $V_i = R + \frac{1}{SC}$

$\therefore H(\omega) = \frac{V_o}{V_i} = \frac{1/SC}{R + 1/SC}$

$\therefore H(\omega) = \frac{V_o}{V_i} = \frac{1/SC}{\frac{SCR+1}{SC}} = \frac{1}{SC} \times \frac{SC}{SCR+1} = \frac{1}{SCR+1}$

But the cut-off frequency (W_c) is obtained when the magnitude of $H(\omega)$ is set to $\frac{1}{\sqrt{2}}$ thus

$|H(\omega)| = \frac{V_o}{V_i} = \left| \frac{1}{SCR+1} \right|$

equating this to $\frac{1}{\sqrt{2}}$ we thus have

$\frac{1}{\sqrt{2}} = \left| \frac{1}{SCR+1} \right|$

$\frac{1}{\sqrt{2}} = \frac{(\sqrt{1})^2}{(\sqrt{SCR})^2 + (\sqrt{1})^2}$

$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{1+(SCR)^2}}$ substituting $S = j\omega$

$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{1+(j\omega CR)^2}}$ Cross multiply and squaring both sides

giving $\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{1+|\omega^2 C^2 R^2|}}$

$\sqrt{2} = \sqrt{1+|\omega^2 C^2 R^2|}$

$(\sqrt{2})^2 = \left(\sqrt{1+|\omega^2 C^2 R^2|}\right)^2$

$2 = 1 + \omega^2 C^2 R^2$ Collecting like terms

$2 - 1 = \omega^2 C^2 R^2$

$1 = \omega^2 C^2 R^2$ dividing through by C^2R^2

$\frac{1}{C^2R^2} = \omega^2$

$\omega = \frac{1}{RC}$

$\therefore W_c = \frac{1}{RC}$