17/ENG05/012

Emerole Chinaeme Ikenna
MECHATRONICS ENGINEERING
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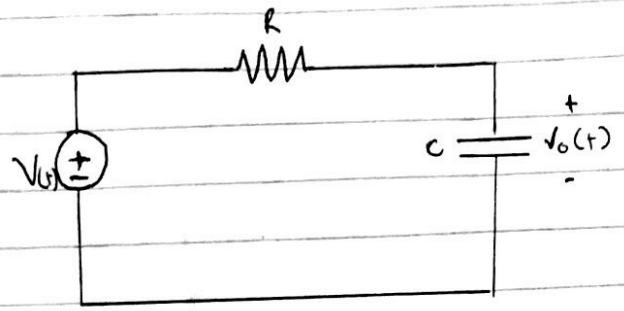
Determine the type of filter shown below & Show that its cut off frequency is

$$\omega_c = 1/RC$$



Solution

A typical low pass filter is shown above; the output is taken off the capacitor

$$\text{Recall } H(\omega) = \frac{V_o}{V_i} = \left(\frac{1/sC}{R + 1/sC}\right)$$

$H(0) = 1$

$H(\infty) = 0$

The cut off frequency ω_c is obtained by setting the magnitude of $H(\omega)$ to $1/\sqrt{2}$

$$|H(\omega)| = \left|\frac{1}{sRC + 1}\right| = \frac{1}{\sqrt{2}}$$

$$= \frac{\sqrt{1^2}}{\sqrt{1^2 + (W^2RC)^2}} = \frac{1}{\sqrt{2}}$$

$$\frac{1}{\sqrt{1 + |W^2R^2C^2|}} = \frac{1}{\sqrt{2}}$$

$$\sqrt{2} = \sqrt{1 + W^2R^2C^2}$$

Squaring both sides

$$2 = 1 + W^2R^2C^2$$

$$2 - 1 = W^2R^2C^2 = 1$$

$$W^2 = \frac{1}{R^2C^2}$$

$$W_C = 1/RC$$