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College: Engineering
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(1) Find the derivative of the following using first principle (b) $y = \sin(3/x^2)$.

Solution.

(b) $y = \sin\left(\frac{3}{x^2}\right)$; $y = \sin 3x^{-2}$

$\Delta y = \sin 3(x+\Delta x)^{-2}$

$\Delta y + y = \sin 3(x+\Delta x)^{-2}$

$\Delta y = \sin 3(x+\Delta x)^{-2} - y$ where $y = \sin 3x^{-2}$

$\Delta y = \sin 3(x+\Delta x)^{-2} - \sin 3x^{-2}$

Let $3(x+\Delta x)^{-2} = A$ and $3x^{-2} = B$

$\therefore \Delta y = \sin A - \sin B$

and remember $\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\cdot\sin\left(\frac{A-B}{2}\right)$

$\therefore \Delta y = 2\cos\left(\frac{A+B}{2}\right)\cdot\sin\left(\frac{A-B}{2}\right)$

$\frac{A+B}{2} = \frac{3(x+\Delta x)^{-2} + 3x^{-2}}{2}$; $\frac{A-B}{2} = \frac{3(x+\Delta x)^{-2} - 3x^{-2}}{2}$

$\frac{A+B}{2} = \left(\frac{3}{(x+\Delta x)^2} + \frac{3}{x^2}\right)\frac{1}{2}$; $\frac{A-B}{2} = \left(\frac{3}{(x+\Delta x)^2} - \frac{3}{x^2}\right)\frac{1}{2}$

$\frac{A+B}{2} = \left(\frac{3x^2 + 3(x+\Delta x)^2}{x^2(x+\Delta x)^2}\right)\frac{1}{2}$; $\frac{A-B}{2} = \left(\frac{3x^2 - 3(x+\Delta x)^2}{x^2(x+\Delta x)^2}\right)\frac{1}{2}$

$\frac{A+B}{2} = \left(\frac{3x^2 + 3x^2 + 6x\Delta x + 3\Delta x^2}{x^2(x+\Delta x)^2}\right)\frac{1}{2}$; $\frac{A-B}{2}\left(\frac{\cancel{3x^2} - \cancel{3x^2} - 6x\Delta x - 3\Delta x^2}{x^2(x+\Delta x)^2}\right)\frac{1}{2}$

$\frac{A+B}{2} = \left(\frac{6x^2 + 6x\Delta x + 3\Delta x^2}{x^2(x+\Delta x)^2}\right)\frac{1}{2}$; $\frac{A-B}{2}\left(\frac{-6x\Delta x - 3\Delta x^2}{x^2(x+\Delta x)^2}\right)\frac{1}{2}$

$$\Delta y = 2\cos\frac{\left(\frac{6x^2 + 6x\Delta x + 3\Delta x^2}{x^2(x+\Delta x)^2}\right)}{2}\cdot\sin\frac{\left(\frac{-6x\Delta x - 3\Delta x^2}{x^2(x+\Delta x)^2}\right)}{2}$$

Divide through by Δx

$$\frac{\Delta y}{\Delta x} = 2\cos\frac{\left(\frac{6x^2 + 6x\Delta x + 3\Delta x^2}{x^2(x+\Delta x)^2}\right)}{2}\cdot\sin\frac{\left(\frac{-6x\Delta x - 3\Delta x^2}{x^2(x+\Delta x)^2}\right)}{2}$$

$$\frac{\Delta y}{\Delta x} = \frac{2\cos\left(\frac{6x^2+6x\Delta x+3\Delta x^2}{2x^2(x+\Delta x^2)^2}\right)\cdot\sin\left(\frac{-6x\Delta x-3\Delta x^2}{2x^2(x+\Delta x)^2}\right)}{\Delta x}$$

$$\frac{\Delta y}{\Delta x} = \frac{2\cos\left(\frac{6x^2+6x\Delta x+3\Delta x^2}{2x^4+(2x^2\Delta x^2)}\right)\cdot\sin\left(\frac{-6x\Delta x-3\Delta x^2}{2x^4+2x^2\Delta x^2}\right)}{\Delta x}$$

$$\frac{\Delta y}{\Delta x} = 2\cos\left(\frac{6x^2}{2x^4+2x^2\Delta x^2}+\frac{6x\Delta x}{2x^2(x+\Delta x)^2}+\frac{3\Delta x^2}{2x^2(x+\Delta x)^2}\right)$$
$$\cdot\sin\left(\frac{-6x\Delta x}{2x^2(x+\Delta x)^2}-\frac{3\Delta x^2}{2x^2(x+\Delta x)^2}\right)\div\Delta x$$

$$\frac{\Delta y}{\Delta x} = 2\cos\left(\frac{3x^2}{(x+\Delta x)^2}+\frac{3\Delta x}{x(x+\Delta x)^2}+\frac{3\Delta x^2}{2x^2(x+\Delta x)^2}\right)$$
$$\times\sin\left(\frac{-\overset{3}{\cancel{6}}x\Delta x}{\cancel{2}x^2(x+\Delta x)^2}-\frac{3\Delta x^2}{2x^2(x+\Delta x)^2}\right)\div\Delta x$$

$$\frac{\Delta y}{\Delta x} = 2\cos\left(\frac{3}{(x+\Delta x)^2}+\frac{3\Delta x}{x(x+\Delta x)^2}+\frac{3\Delta x^2}{2x^2(x+\Delta x)^2}\right)$$
$$\times\sin\left(\frac{-3\Delta x}{x(x+\Delta x)^2}-\frac{3\Delta x^2}{2x^2(x+\Delta x)^2}\right)\div\Delta x$$

multiply through by 1/2

$$\frac{\Delta y}{\Delta x} = \frac{\frac{1}{\cancel{2}}\times\cancel{2}\cos\left(\frac{3}{(x+\Delta x)^2}+\frac{3\Delta x}{x(x+\Delta x)^2}+\frac{3\Delta x^2}{2x^2(x+\Delta x)^2}\right)\cdot\sin\left(\frac{-3\Delta x}{x(x+\Delta x)^2}-\frac{3\Delta x^2}{2x^2(x+\Delta x)^2}\right)}{\Delta x/2}$$

$$\frac{\Delta y}{\Delta x} = \frac{\cos\left(\frac{3}{(x+\Delta x)^2}+\frac{3\Delta x}{x(x+\Delta x)^2}+\frac{3\Delta x}{2x^2(x+\Delta x^2)}\right)\cdot\sin\left(\frac{-3\Delta x}{x(x+\Delta x)^2}-\frac{3\Delta x}{2x^2(x+\Delta x)}\right)}{\Delta x/2}$$

$$\lim_{\Delta x \to 0} \frac{dy}{dx}\ 2\cos\left(\frac{3}{(x)^2}+\frac{0}{0}+\frac{0}{0}\right)\cdot\sin\left(\frac{0}{0}-\frac{0}{0}\right)$$

$$\therefore \frac{dy}{dx} = \cos\frac{3}{x^2}$$

(6) $y = 4/x^3$

Solution

$y = 4(x)^{-3}$

$\Delta y = 4(x+\Delta x)^{-3}$

$\Delta y = \frac{4}{(x+\Delta x)^3}$

$\Delta y + y = \frac{4}{(x+\Delta x)^{3}}$

$\Delta y = \frac{4}{(x+\Delta x)^3} - y$ where $y = 4x^{-3}$

$\Delta y = \frac{4}{(x+\Delta x)^3} - \frac{4}{x^3}$

$\Delta y = \frac{4x^3 - 4(x+\Delta x)^3}{x^3(x+\Delta x)^3}$

$\Delta y = \frac{4x^3 - 4(x^3 + 3x^2\Delta x + 3x\Delta x^2 + \Delta x^3)}{x^3(x^3 + 3x^2\Delta x + 3x\Delta x^2 + \Delta x^3)}$

$\Delta y = \frac{\cancel{4x^3} - \cancel{4x^3} - 12x^2\Delta x - 12x\Delta x^2 - 4\Delta x^3}{x^3(x^3 + 3x^2\Delta x + 3x\Delta x^2 + \Delta x^3)}$

$\therefore \Delta y = \frac{-12x^2\Delta x - 12x\Delta x^2 - 4\Delta x^3}{x^3(x^3 + 3x^2\Delta x + 3x\Delta x^2 + \Delta x^3)}$

Let $x^3 + 3x^2\Delta x + 3x\Delta x^2 + \Delta x^3 = t$

$\therefore \Delta y = \frac{-12x^2\Delta x - 12x\Delta x^2 - 4\Delta x^3}{x^3 t}$

$\therefore \Delta y = \frac{-12\cancel{x^2}\Delta x}{x^3 t} - \frac{12\cancel{x}\Delta x^2}{x^2 t} - \frac{4\Delta x^3}{x^3 t}$

$\Delta y = -\frac{12\Delta x}{xt} - \frac{12\Delta x^2}{x^2 t} - \frac{4\Delta x^3}{x^3 t}$

Divide through by Δx

$\frac{\Delta y}{\Delta x} = \frac{-12\Delta x}{xt\Delta x} - \frac{12\Delta x^2}{x^2 t\Delta x} - \frac{4\Delta x^2}{x^3 t \Delta x}$

$\therefore \frac{\Delta y}{\Delta x} = \frac{-12}{x(x^3 + 3x^2\Delta x + 3x\Delta x^2 + \Delta x^3)} - \frac{12}{x^2(x^3 + 3x^2\Delta x}$

$\frac{12}{x^2(x^3 + 3x^2\Delta x + 3x\Delta x^2 + \Delta x^3)} - \frac{4\Delta x^2}{x^3(x^3 + 3x^2\Delta x + 3x\Delta x^2 + \Delta x^3)}$

lim $\Delta x \to 0$ $\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{-12}{x(x^3 + 0 + 0 + 0)} - 0 - 0$

$\frac{dy}{dx} = \frac{-12}{x^4}$

$\therefore \frac{dy}{dx} = -12x^{-4}$ //

12) Find the integral of the following

(a) $\frac{dx}{x^2+36}$

solution

$$\int \frac{1}{x^2+6^2}\,dx = \int \frac{dx}{x^2+6^2}$$

$x = 6\tan\theta$

$\frac{dx}{d\theta} = 6\sec^2\theta$, $dx = 6\sec^2\theta\, d\theta$

$x^2+6^2 = 6\tan^2\theta + 6^2 = 6^2(\tan^2\theta + 1)$

factorising after substituting

$$\int \frac{6\sec^2\theta\, d\theta}{36\sec^2\theta} = \int \frac{d\theta}{6} = \frac{1}{6}\int d\theta = \frac{1}{6}\theta) + c$$

but $\theta = \tan^{-1} x/6$

$\therefore \frac{1}{6}\tan^{-1}\frac{x}{6} + c$ is the integral of $\frac{dx}{(x^2+36)}$

(b) $\frac{dx}{(x^2+13)}$

solution.

$$\int \frac{dx}{x^2+\sqrt{13}} = \int \frac{1}{x^2+\sqrt{13}}\,dx$$

$x = \sqrt{13}\tan\theta$ $\quad \frac{dx}{d\theta} = \sqrt{13}\sec^2\theta$ $\quad \therefore dx = d\theta\sqrt{13}\sec^2\theta$

$x^2+\sqrt{13} = \sqrt{13}\tan^2\theta + \sqrt{13} = \sqrt{13}(\tan^2\theta + 1)$

by factorisation.

substitute.

$$\int \frac{dx}{x^2+13} = \int \frac{d\theta\sqrt{13}\sec^2\theta}{13\sec^2\theta} = \int \frac{d\theta}{\sqrt{13}} = \frac{1}{\sqrt{13}}\int d\theta$$

$$= \frac{1}{\sqrt{13}}(\theta) + c = \frac{1}{\sqrt{13}}(\theta) + c$$

but $\theta = \tan^{-1}\frac{x}{\sqrt{13}}$

$$\frac{1}{\sqrt{13}}\tan^{-1}\frac{x}{\sqrt{13}} + c$$